Math 130 04 – A Survey of Calculus

Homework assignment 2

Due: Tuesday, September 13, 2022

Write your answers on a separate sheet of paper. Write your name at the top of each page
you use, and number each page. Remember to number your answers correctly.
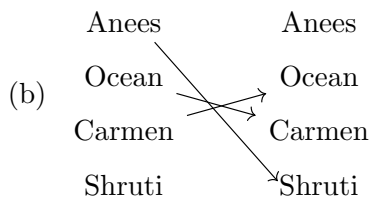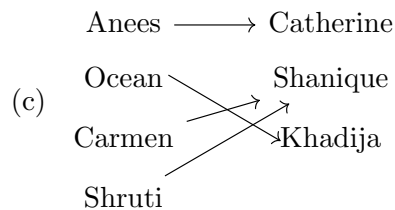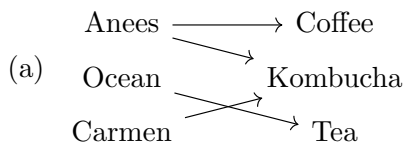

Sets and functions

1. Consider the sets

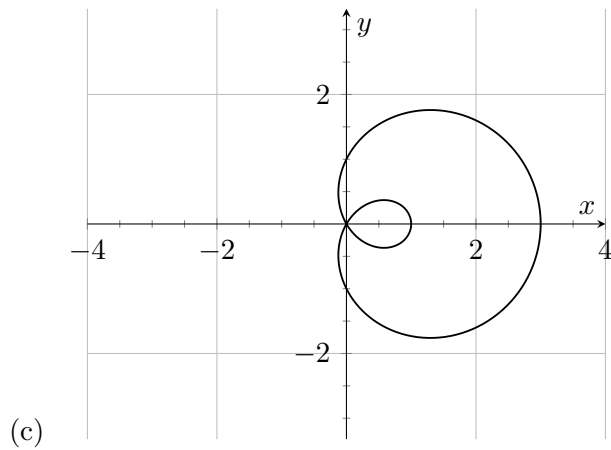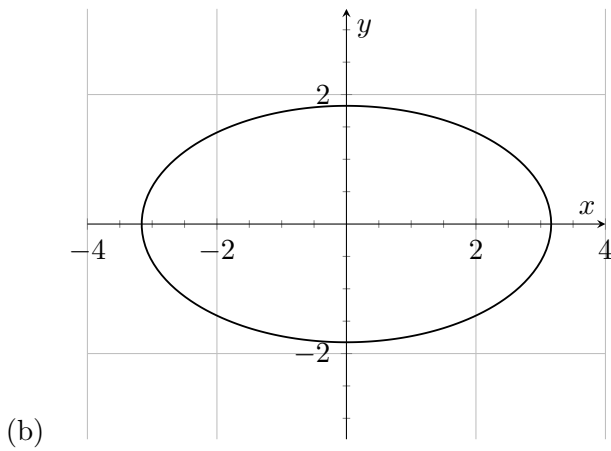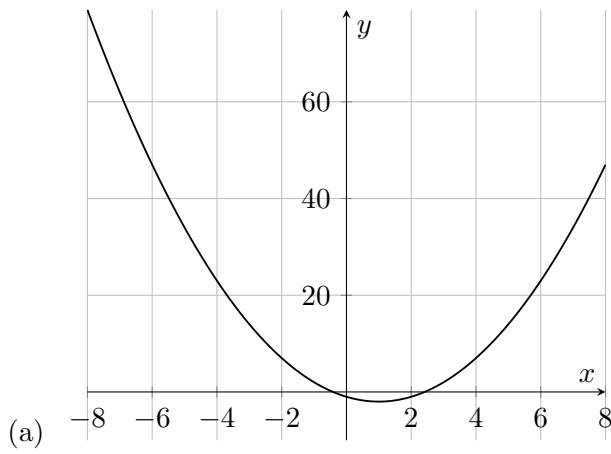$$A = \{1, 3, 5, 7, 9\} \quad B = \{2, 4, 6, 8\} \quad C = \{1, 2, 3, 7, 8, 9\}.$$

   What are the following sets? Justify your answers.

   (a) $A \cap B$ (The intersection of A and B.)

   (b) $A \cap B \cap C$

   (c) $A \cup (B \cap C)$

   (d) $(A \cup B) \cap C$

2. Write the domain and the range of each of the following correspondences. Which of the following correspondences are functions? Justify your answers.

(a)
Anees ⟶ Coffee
Ocean   Kombucha
Carmen ⟶ Tea

(c)
Anees ⟶ Catherine
Ocean   Shanique
Carmen   Khadija
Shruti

(b)
Anees   Anees
Ocean   Ocean
Carmen   Carmen
Shruti   Shruti

Graphs

3. Which of the following graphs are functions? Justify your answers.

(a)

(b)

(c)

4. A new tech manufacturer, Lemon Inc., released a phone (the piePhone) in 2022 at a price point of $500. The piePhone was a success, and by September, the 2 million units of the piePhone produced in 2022 had sold out. In 2023, Lemon plans to release the piePhone2 (a minor upgrade to the piePhone). To decide how to price the piePhone2, Lemon has studied the market.
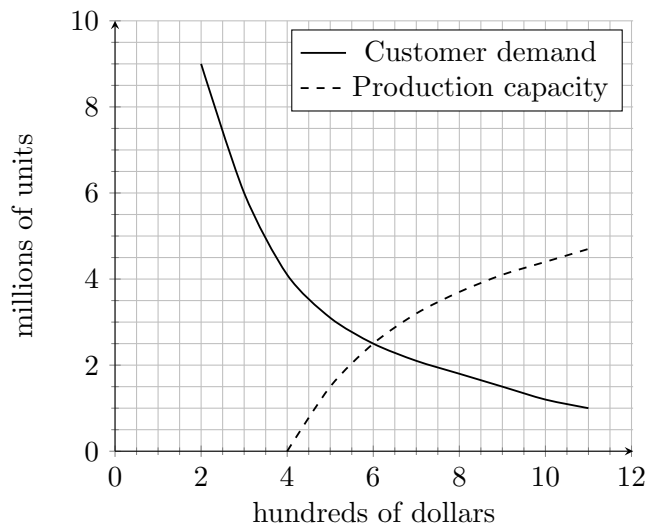

Figure 1: Market study

The result of the study (the "Customer demand" graph above) shows the number of units of the piePhone2 that customers will buy at a given price point. For example, if the piePhone2 is priced at $300, then Lemon can expect to sell 6 million units, but if it is priced at $900, then only 1.5 million units will sell. On top of the market study, Lemon has drawn a graph ("Production capacity") showing how many units they can produce at a given price point per unit. For instance, if the piePhone2 is priced at less than $400, then Lemon can't afford to produce it, but if it is priced at $1000, then Lemon can produce about 4.5 million units in 2023.

(a) Suppose Lemon produces 2 million units of the piePhone2 in 2023.
   i. What is the lowest price that Lemon can afford to sell the piePhone2 at?
   ii. How many units can Lemon expect to sell?
   iii. What will the final price per unit be?
   iv. What will Lemon's revenue (number of units sold × price per unit) be?

(b) Suppose Lemon produces 4 million units of the piePhone2 in 2023.
   i. What is the lowest price that Lemon can afford to sell the piePhone2 at?
   ii. How many units can Lemon expect to sell?
   iii. What will the final price per unit be?
   iv. What will Lemon's revenue (number of units sold × price per unit) be?

(c) How many units of the piePhone2 should Lemon produce in 2023 so that every unit will sell? What will the final price per unit be? What will Lemon's revenue be in 2023?