# Math 130 04 – A Survey of Calculus

Homework assignment 6

Due: Tuesday, October 18, 2022

**Instructions:** Write your answers on a separate sheet of paper. Write your name at the top of each page you use, and number each page. Number your answers correctly.
**Justify all your answers.**

**Recall:** The derivatives of some standard functions are:

- If $f(x) = a$ for any fixed real number $a$ (i.e. $f$ is a constant function), then $f'(x) = 0$.
- If $f(x) = x$, then $f'(x) = 1$.
- If $f(x) = x^a$, then $f'(x) = a \cdot x^{a-1}$ (for any fixed real number $a$).

1. Recall the rules for derivatives: Let $f, g$ be real functions that are differentiable over an interval $I$. Then,

   - **Sum rule:**
   $$(f+g)'(x) = f'(x) + g'(x)$$
   - **Product rule:**
   $$(f \cdot g)'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$
   - **Quotient rule:** If $g(x)$ is never 0 over $I$, then
   $$\frac{f}{g}(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$$

   Using these, evaluate the derivatives of the following functions (over any interval where they are defined).

   (a) $f(x) = x^3 - 9x^2 + 16$ (b) $f(x) = \dfrac{x^3 + 25}{3x - 2}$ (c) $f(x) = \dfrac{x+3}{x^2 - 4}$

2. The distance between Los Angeles and San Diego on the I-5 highway is 118 miles.
   (a) What is the average speed (in mph) required to do the trip in 1.5 hours?
   (b) If the speed limit is 70 mph all along the I-5, is it possible to do the trip in 1.5 hours without breaking the law? Explain. What about in 2 hours?
   (c) A car going from L.A. to San Diego on the I-5 travels $f(x)$ miles (measured from L.A.) after $x$ hours, where $f$ is the function:
   $$f(x) = x(91 - 16x)$$
      i. How long does the car take to reach San Diego (i.e. cover 118 miles)?
      ii. Does the car ever break the law? (Is its speed ever more than 70 mph?)
      iii. What is the car's speed when it leaves L.A. (at the starting time)? What is the car's speed when it arrives in San Diego? (That is, at the time calculated in part i.)

   **Remember:** Speed (or velocity) is the derivative of distance as a function of time.

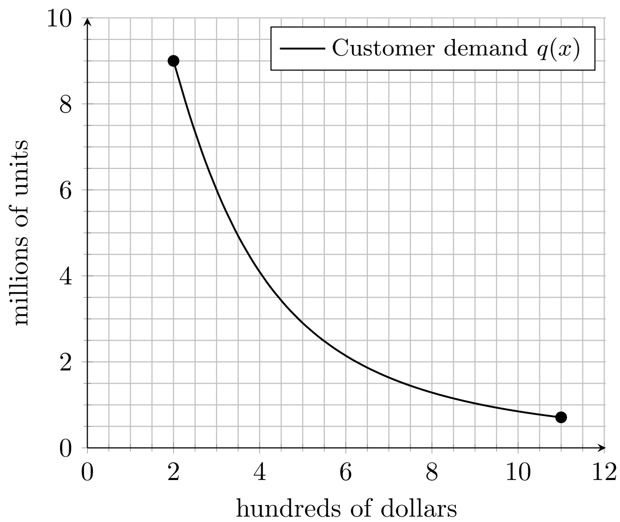


Figure 1: Market demand model

3. (The return of Lemon, Inc. ...) In Homework 2, we saw Lemon Inc.'s market study that estimated the number of units of the piePhone2 that they could expect to sell at a given price point (the "customer demand" graph). Lemon can see that customer demand ($q(x)$ millions of units sold at a price per unit of $x$ hundred dollars) is a continuous function (over the interval $[2, 11]$), since it satisfies the vertical line test and the pen-to-paper test.

   Lemon also know that their *revenue* from selling $q(x)$ million units at $x$ hundred dollars each is $x \cdot q(x)$ hundred million dollars (Hmn\$). That is, their revenue function is:

   $$r(x) = x \cdot q(x) \quad \text{(in Hmn\$)}$$

   (a) Lemon wants to know the *rate of change* (i.e. the derivative) of revenue in terms of the rate of change (i.e. the derivative) of demand. Show that we can write the rate of change of revenue as:

   $$r'(x) = q(x) \cdot \left(1 + \frac{x}{q(x)} \cdot q'(x)\right)$$

   **Remark:** The function $E_d(x) = \dfrac{x}{q(x)} \cdot q'(x)$ is called the *"price elasticity of demand"* (Wikipedia article) (that some of you may have seen in your other classes). It is extremely important in economics — it measures how sensitive demand is to changes in price. $E_d(x)$ is almost always a negative real number (i.e. $E_d(x) < 0$). If $E_d(x) = -2$, it means that a 10% increase in price will result in a 20% *decrease* in demand.*

   (b) Lemon hires some pretty solid economists who figure out that the demand function $q$ is given by:

   $$q(x) = \frac{90}{x^2 + 6}$$

   i. Calculate $q'(x)$ and $r'(x)$.
   ii. Calculate $r'(2)$. Is revenue increasing or decreasing at a price point of \$200 per unit?
   iii. Calculate $r'(6)$. Is revenue increasing or decreasing at a price point of \$600 per unit?
   iv. What is $E_d(6)$? At a price point of \$600 per unit, how much (in %) will demand increase or decrease if the price increases by 7%?

*Now that you understand derivatives, you know the exact definition of the price elasticity of demand.