

Math 130 04 – A Survey of Calculus

Homework assignment 7

Due: Tuesday, November 1, 2022

Remember: The **chain rule** says that if f and g are real functions, then the derivative of the composite function $(g \circ f)(x) = g(f(x))$ can be calculated as follows.

$$(g \circ f)'(x) = g'(f(x)) \cdot f'(x)$$

1. Evaluate the derivatives of the following functions.

(a) $f(x) = (x^2 + 3x + 4)^3$

(b) $f(x) = (x^3 + 5)^{1/4}$

(c) $f(x) = \frac{x + 3}{(x^2 - 4)^{2/3}}$

2. (Variable interest rates.)

A lender (e.g. a bank) proposes two 20-year loans of \$1.5 million to a borrower (e.g. a homebuyer). The borrower can either pay *compound* interest at an interest rate of x percent, or *simple* interest at an interest rate of $2x$ percent. So, if the borrower chooses to pay compound interest at a rate of x percent, the amount they have to pay after 20 years is

$$f(x) = 1.5 \cdot \left(1 + \frac{x}{100}\right)^{20} \text{ million dollars.}$$

If the borrower chooses to pay simple interest, the amount they have to pay after 20 years is

$$g(x) = 1.5 \cdot \left(1 + 20 \cdot \frac{2x}{100}\right) \text{ million dollars.}$$

(a) If the compound interest rate is 4 percent, how much would the borrower pay after 20 years if they chose the loan with:

- i. Compound interest.
- ii. Simple interest.

Which loan makes more sense for the homebuyer?

(b) If the compound interest rate is 8 percent, how much would the borrower pay after 20 years if they choose the loan with:

- i. Compound interest.
- ii. Simple interest.

Which loan makes more sense for the homebuyer?

(c) The difference between the two loans is measured by the amount $d(x) = g(x) - f(x)$.

- i. Calculate the rate of change of the difference between the two loans at a compound interest rate of 2 percent.
- ii. Calculate the rate of change of the difference between the two loans at a compound interest rate of 4 percent.
- iii. At what compound interest rate is the difference between the two loans *maximum*?