# Math 130 04 – A Survey of Calculus

Homework assignment 8

Due: Tuesday, December 6, 2022

**Remember:** If $f$ is a continuous real function, the *Riemann integral* of $f$ is the function

$$\left(\int f\right)(x) = \int_0^x f(x) \cdot dx = \lim_{h \to 0} S_h f(x)$$

**Remember:** The *fundamental theorems of calculus* are the following facts.

**FTC1** If $f$ is a differentiable real function, then

$$\boxed{\left(\int f'\right)(x) = \int_0^x f'(x) \cdot dx \ = \ f(x) - f(0)}$$

**FTC2** If $f$ is a continuous real function, then

$$\boxed{\left(\int f\right)'(x) = f(x)}$$

**Remember:**

- If $f(x) = x^a$ where $a$ is any constant real number such that $a \neq -1$, then

$$\left(\int f\right)(x) = \int_0^x x^a \cdot dx \ = \ \frac{x^{a+1}}{a+1}$$

- If $f(x) = e^{kx}$ where $k$ is any constant real number such that $k \neq 0$, then

$$\left(\int f\right)(x) = \int_0^x e^{kx} \cdot dx \ = \ \frac{e^{kx} - 1}{k}$$

**Rules for Riemann integrals**

- **Constant rule:** If $f(x) = c \cdot g(x)$, where $c$ is any constant real number, then

$$\left(\int f\right)(x) = c \cdot \left(\int g\right)(x)$$

  or using alternate notation,

$$\int_0^x c \cdot g(x) \cdot dx = c \cdot \int_0^x g(x) \cdot dx$$

- **Sum rule:** If $f$ and $g$ are continuous real functions, then

$$\left(\int (f+g)\right)(x) = \left(\int f\right)(x) + \left(\int g\right)(x)$$

  or using alternate notation,

$$\int_0^x (f(x) + g(x)) \cdot dx = \left(\int_0^x f(x) \cdot dx\right) + \left(\int_0^x g(x) \cdot dx\right)$$

1. Calculate the Riemann integrals of the following functions.
   (a) $f(x) = x^2 + 3x + 4$
   (b) $f(x) = 8e^{-2x} + 4$
   (c) $f(x) = 4x^{-1/3}$
   (d) $f(x) = 3x(4 + x^{2/5})$

**Recall:** The *marginal cost function* is the derivative of the total cost function. Similarly, the marginal revenue and marginal profit functions are the derivatives of the total revenue and total profit functions respectively.

2. A company calculates its marginal cost function $C'$ as follows: If $x$ thousand units have been produced, the marginal cost (i.e. the cost to produce the next unit) is

$$C'(x) = 2x^{-1/3} \qquad \text{dollars per unit.}$$

   (a) Find the company's total cost function $C$ (i.e. $C(x)$ thousands of dollars to produce $x$ thousand units) if the fixed cost to produce 0 units is 3 thousand dollars (i.e. $C(0) = 3$).
   (b) Suppose the company's marginal revenue function is as follows: If $x$ thousand units have been sold, the marginal revenue (i.e. the revenue from selling the next unit) is

$$R'(x) = 3x^{-1/2} \qquad \text{dollars per unit.}$$

      i. Find the marginal profit function $P'$. (Remember that the total profit function $P$ is defined as $P(x) = R(x) - C(x)$, where $R$ and $C$ are the total revenue and total cost functions.)
      ii. Does the total profit function $P$ have a maximum in the interval $[0, 20]$? If so, find the value $a \in [0, 20]$ such that $P$ has a maximum at $a$.
      iii. Calculate the total profit function $P$, assuming that the revenue from selling 0 units is 0 dollars (i.e. $R(0) = 0$).

**Recall:** If $f$ is a continuous function, the value of the Riemann integral of $f$ at $x$, i.e. the real number $(\int f)(x)$, is the area "under" the graph of $f$ between the points 0 and $x$ on the horizontal axis.

3. Consider the following definition.

$$f(x) = \begin{cases} x^5 + 2x^2 - 2 & \text{if } x \leq 0 \\ \\ x^3 + 4x - 2 & \text{if } x > 0 \end{cases}$$

   (a) Is the function $f$ continuous? Namely, is $f$ continuous at every real number $x$?
   (b) What is the value of $(\int f)(x)$ if $x \leq 0$ ?
   (c) What is the value of $(\int f)(x)$ if $x > 0$?