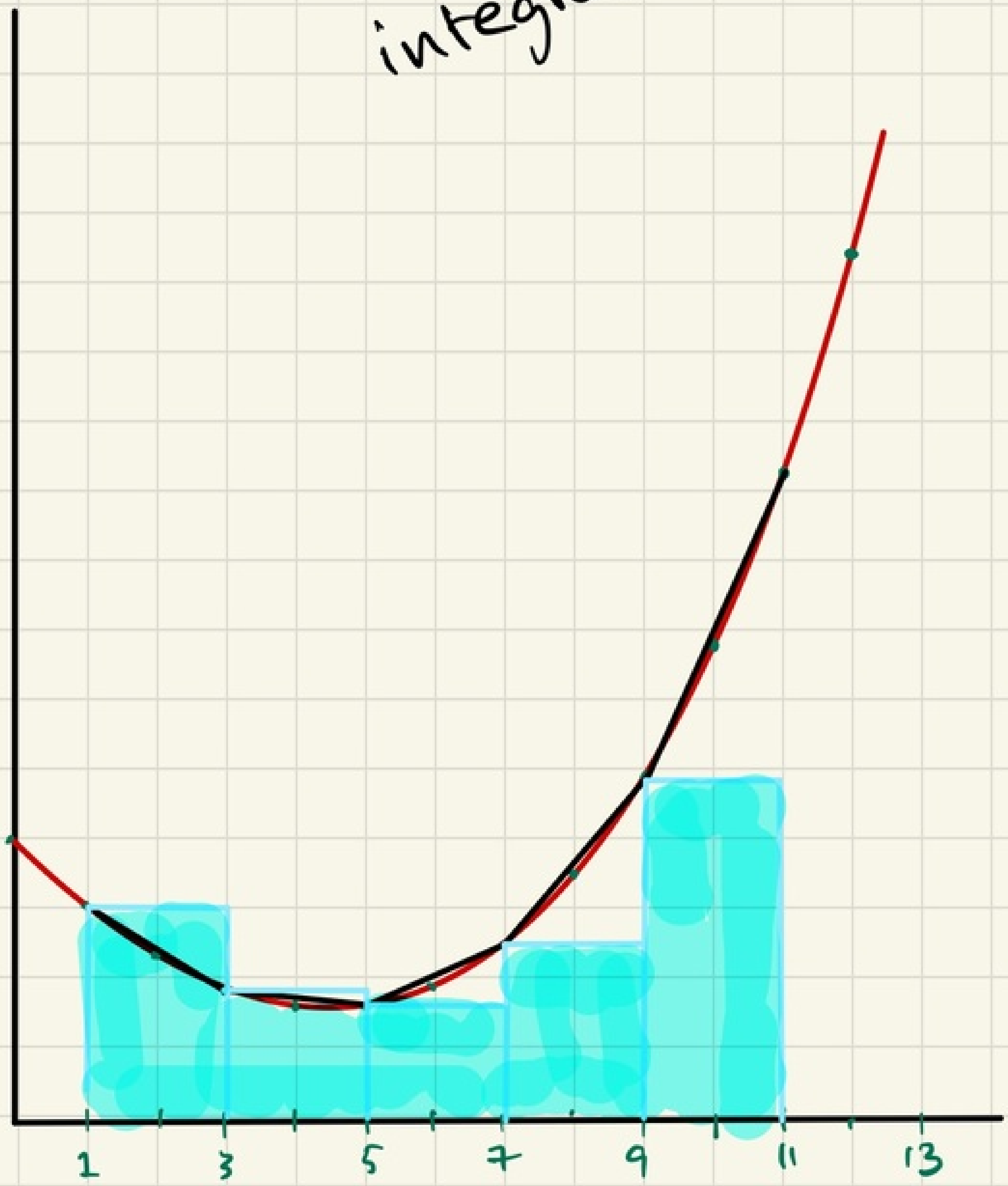
Lecture 9

"derivatives"

Rates of change, $\underbrace{\text{difference quotients}}$,

$\underbrace{\text{RIEMANN}}_{\text{(lower)}}$ sums, Fund. theorem of calc.

integrals



Application: Interest rates

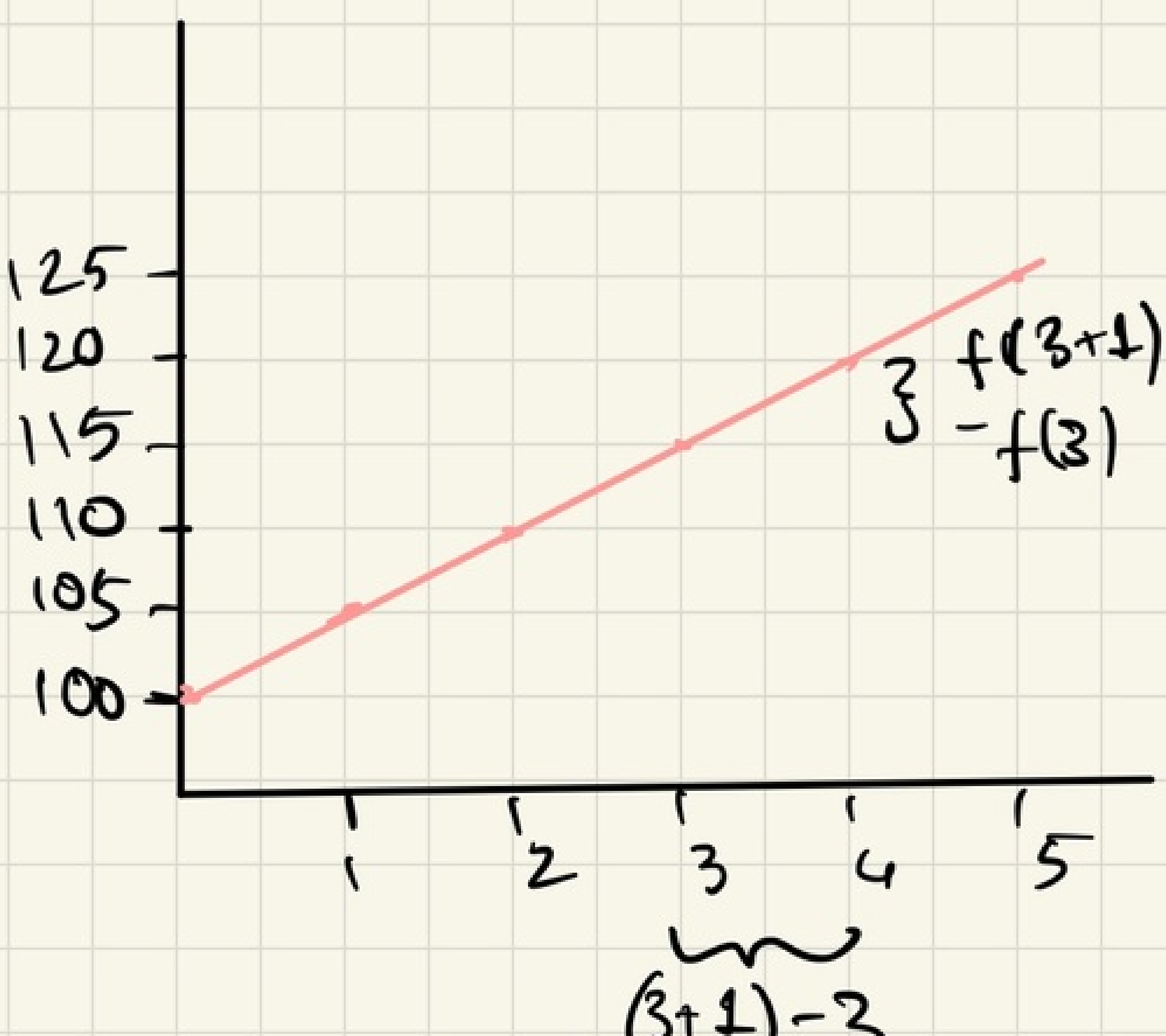
You put \$100 in an acct at a bank

The bank promises that after

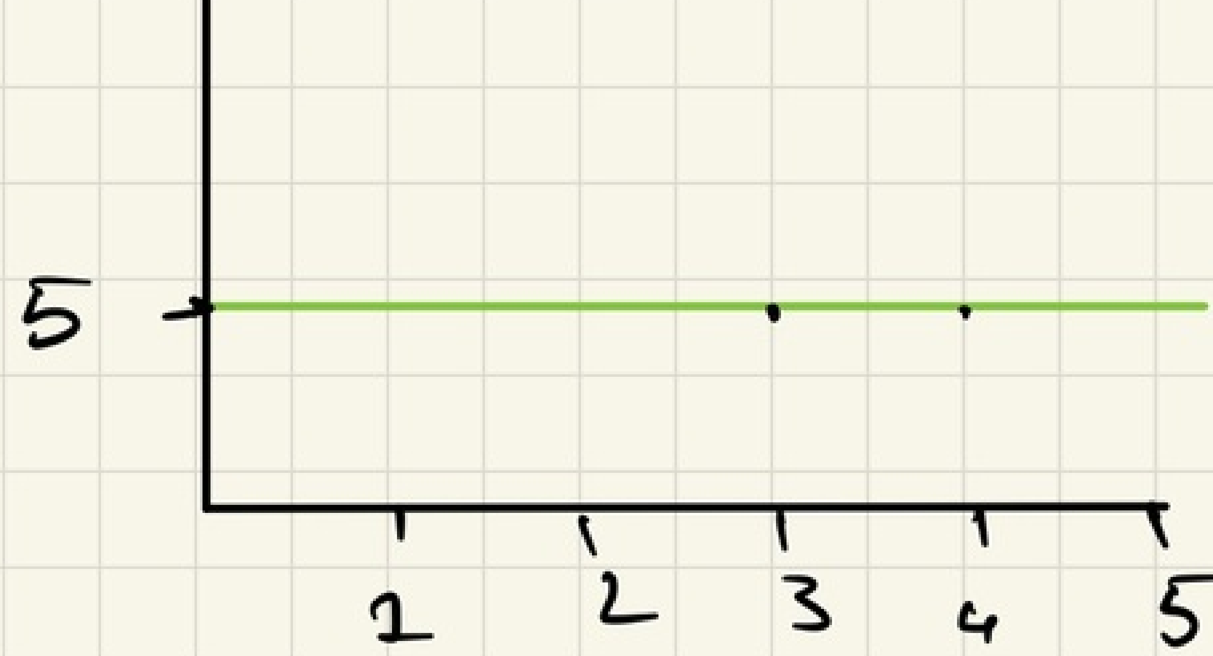
Years	Balance
1	105
2	110
3	115
4	120
5	125

Q: What's the interest rate? (on \$100)

Q: Does it fluctuate?



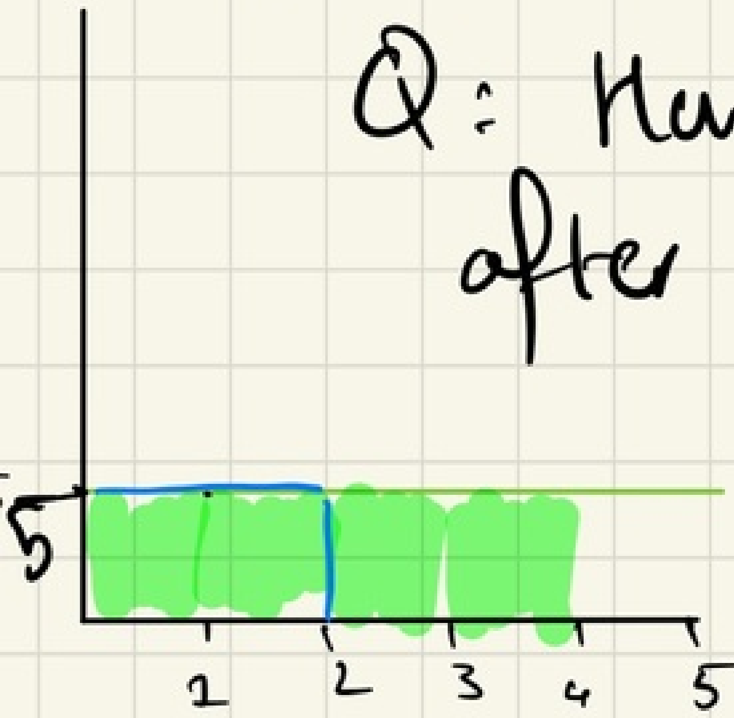
From the "balance" function, you know the interest/ yearly payout function

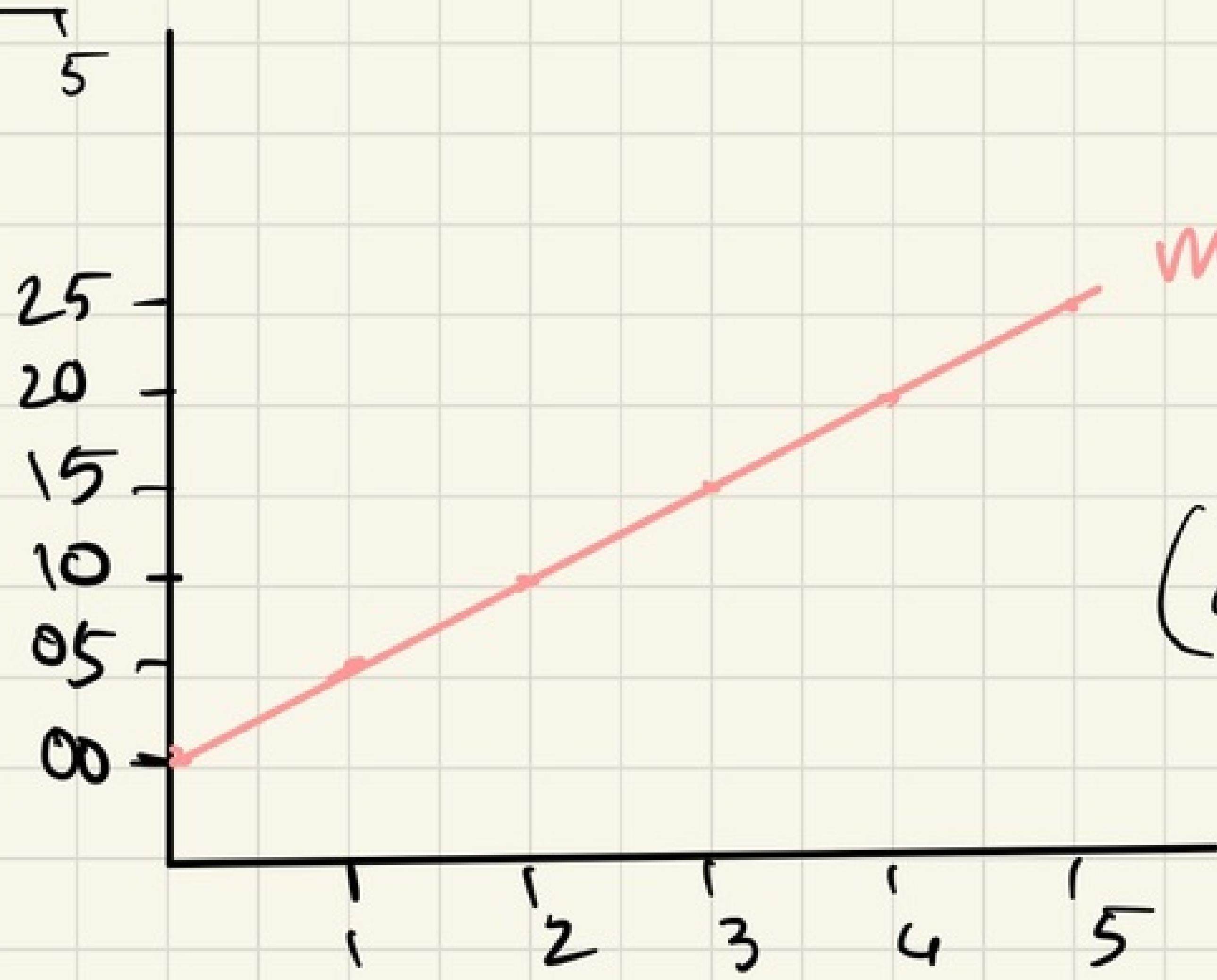


Your parents/someone puts a certain secret amount of money in an acct. The bank gives you \$5/year.

Q: How much money have you earned after

Years	1	2	3	4	5
Amount	5	10	15	20	25





money earned function

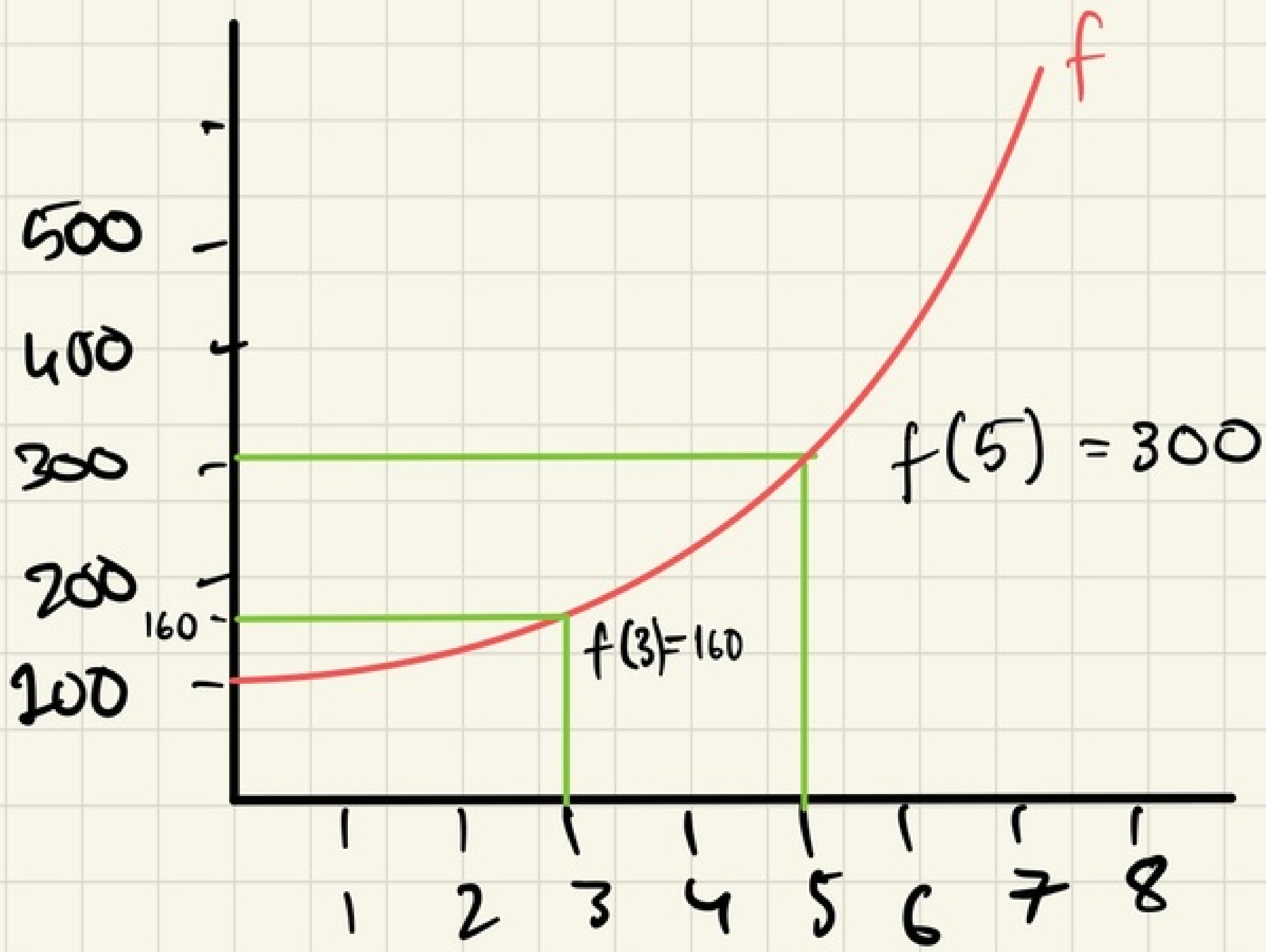
(acct balance = money earned + secret amt)

from the payout function, you know the acct. balance up to a constant

acct balance (x) = money earned (x) + c_0 (initial amount)

Rate of change

E.g. Deer population:

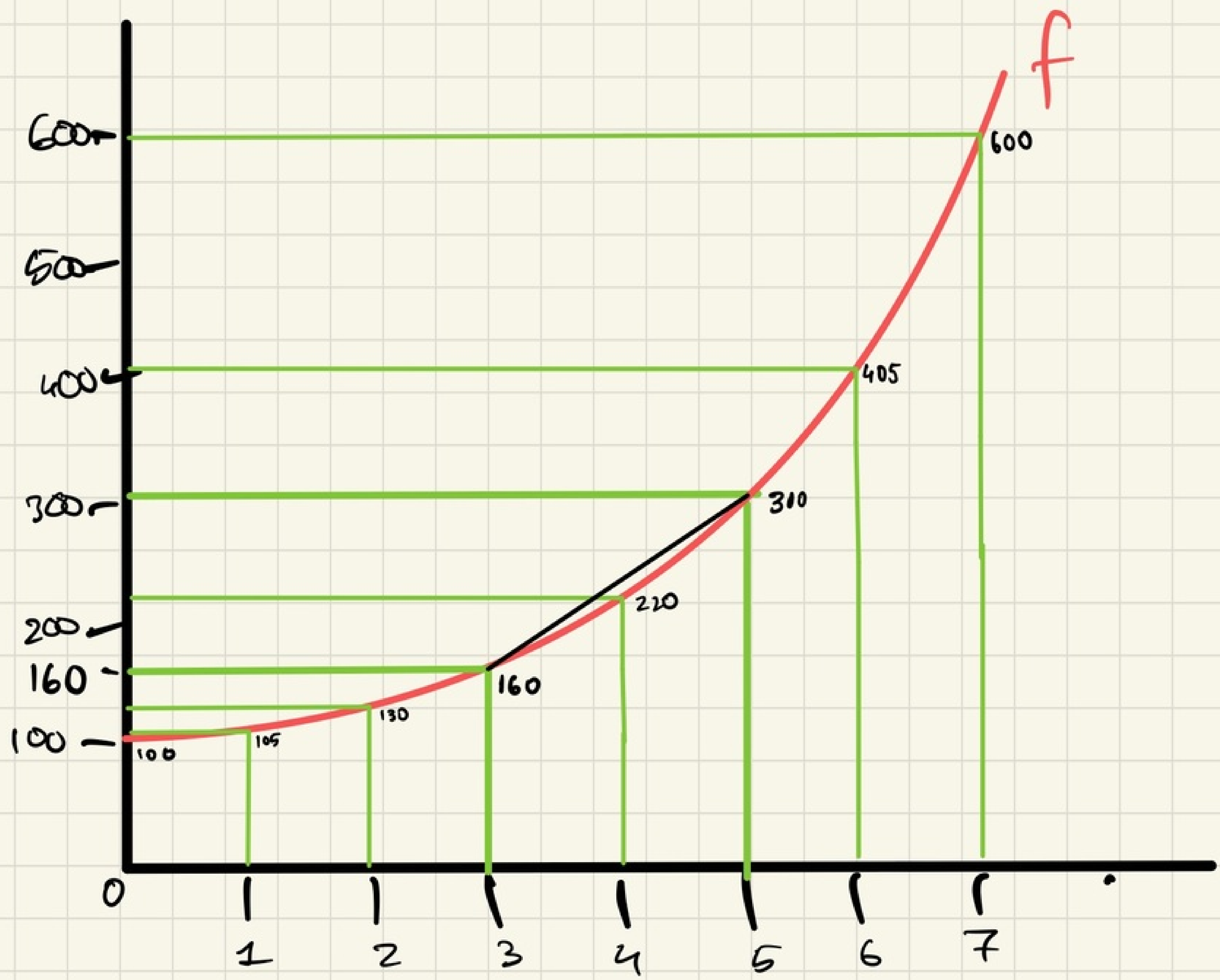


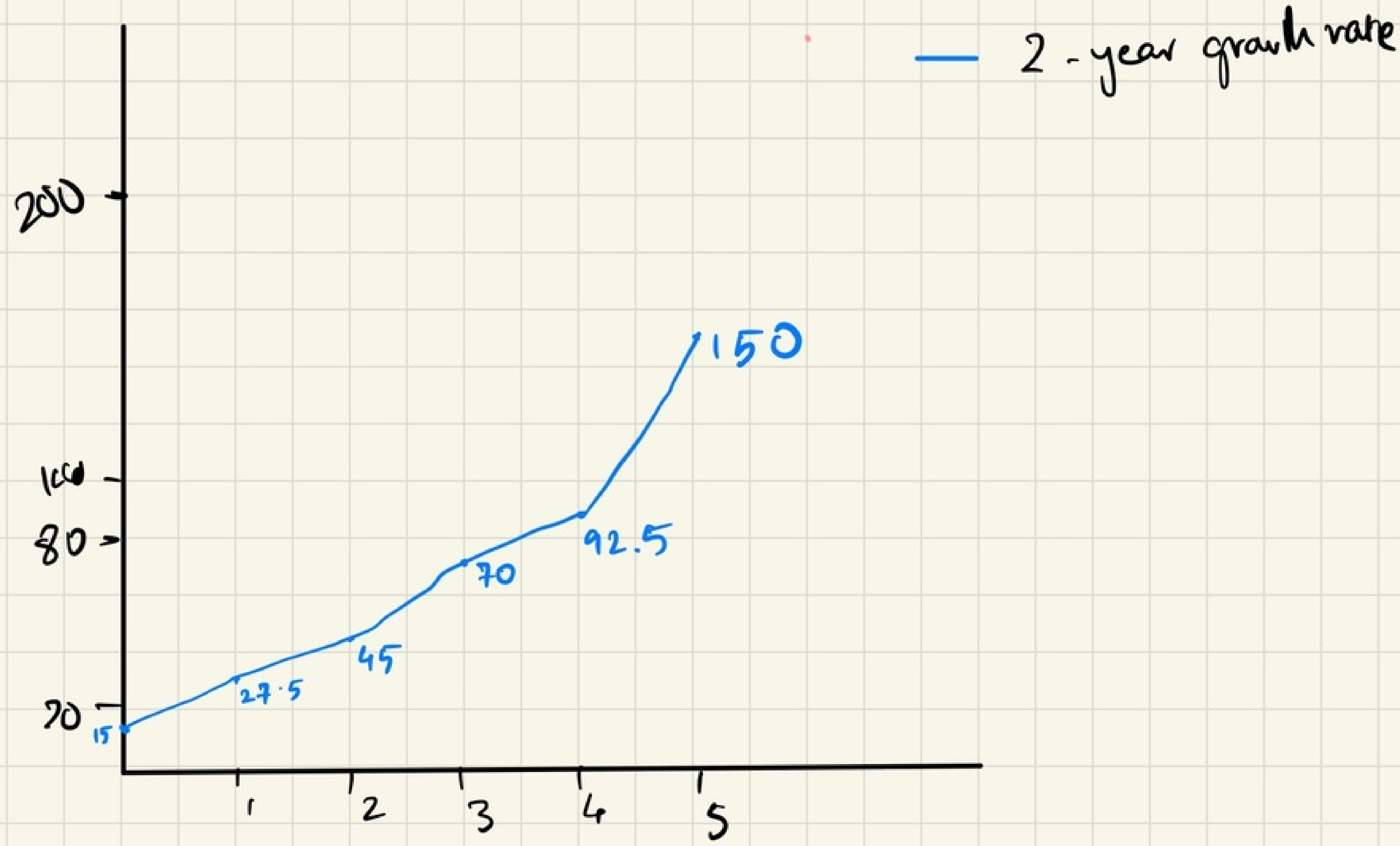
From years 3 to 5,

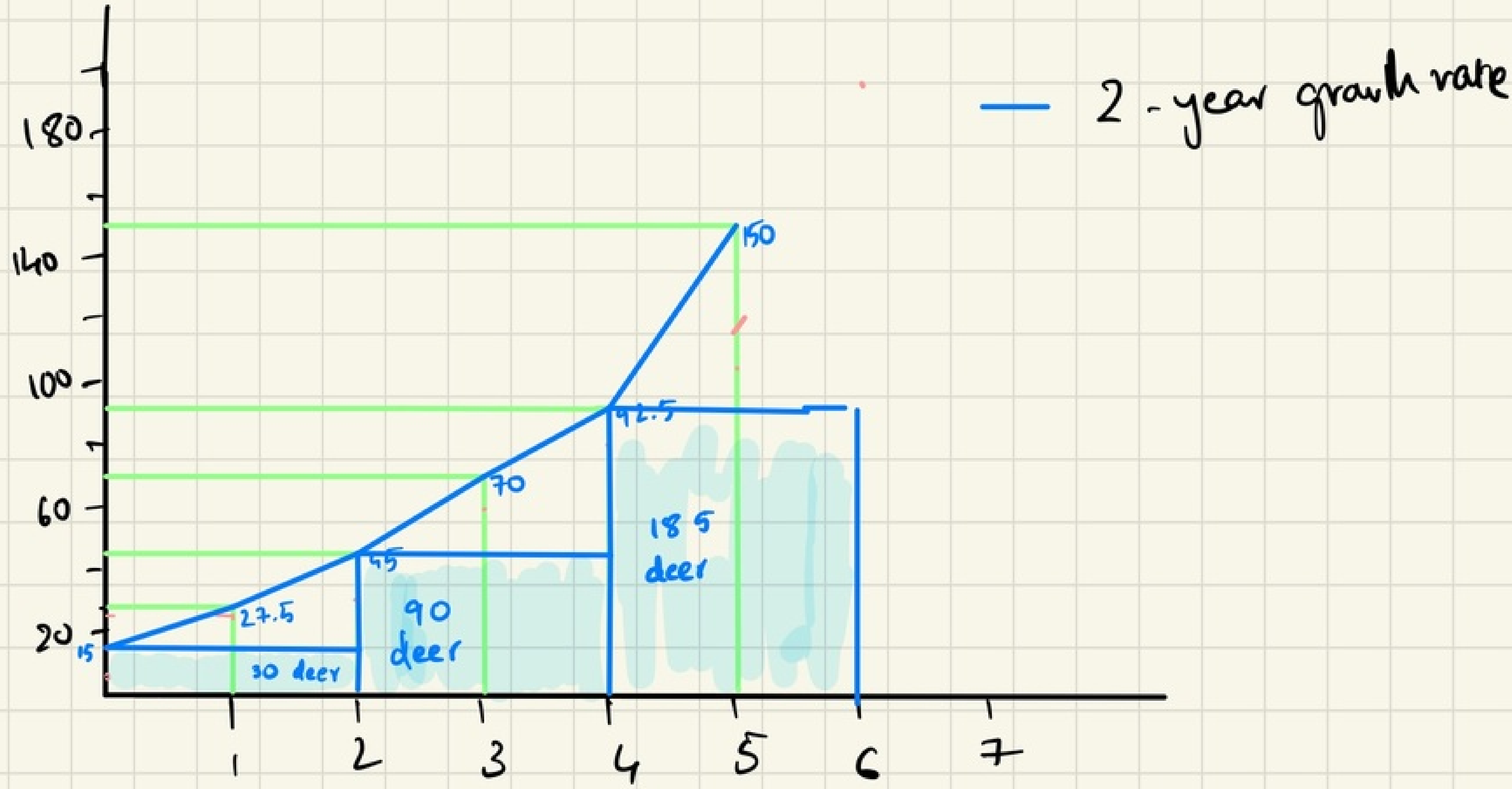
$$f(3+2) - f(3) = 300 - 160 = 140$$

so the rate of change of f over $[3,5]$ was 70 deer/yr

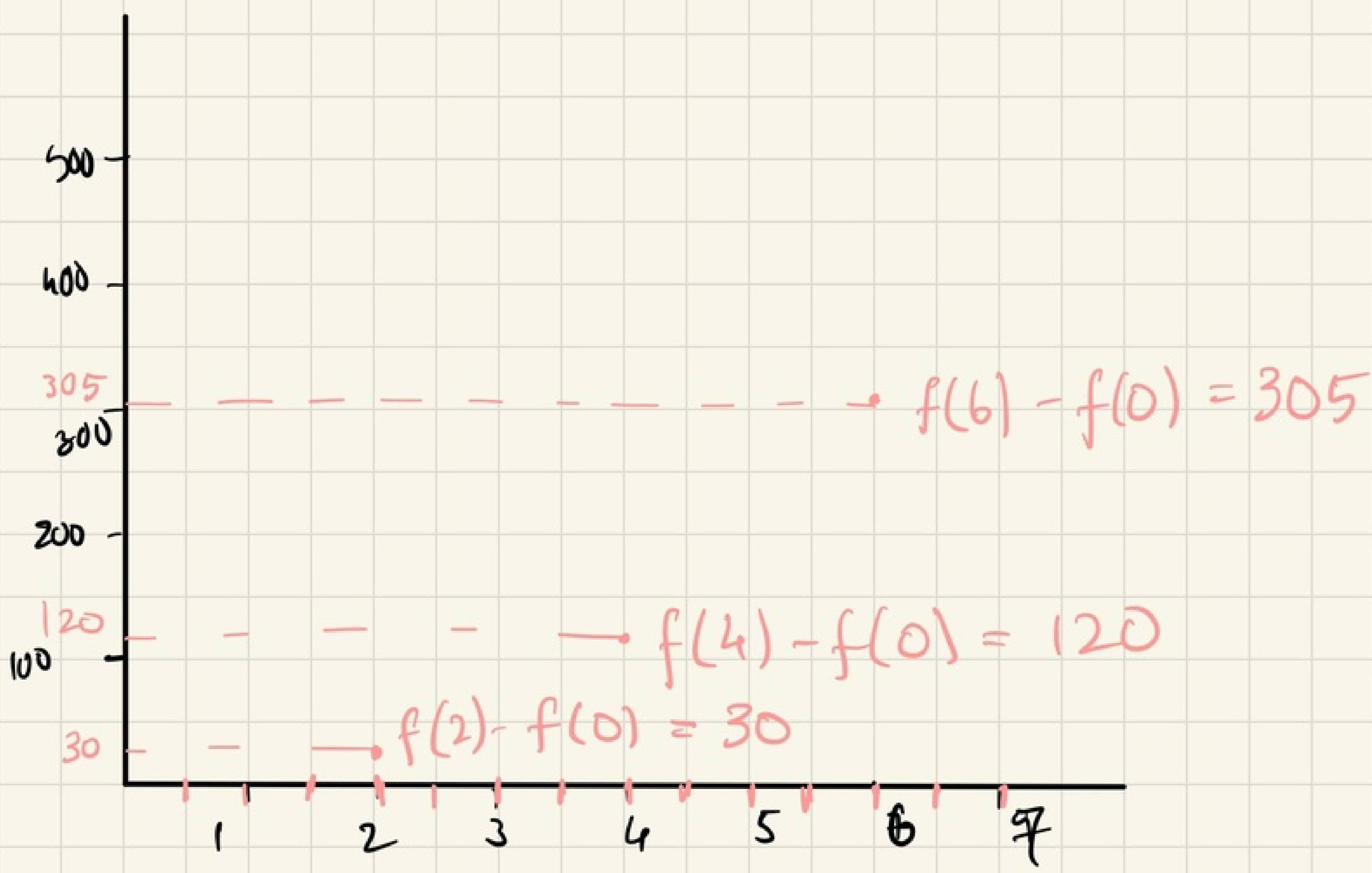
The "2-year growth rate of f at 3" was 70 deer per year


f
600
500
400
300
200
160
100
0
1
2
3
4
5
6
7
100
105
130
160
220
310
405
600


2 - year growth rate
200
100
80
20
15
27.5
45
70
92.5
150
1
2
3
4
5



- Suppose we only know the two year growth rate.
- We still know how much the population has grown every 2 years



(h-) Difference quotients & (h-) Riemann sums

Let f be some continuous function (on $\mathbb{R}$)

▷ The (h-) difference quotient of f is the function

$$Df(x) = \frac{f(x+h) - f(x)}{h}$$

$(h = x + h - x)$

▷ The h-Riemann sum of f is the function

$$Sf(x) = \begin{cases} h \cdot \left(f(0) + f(h) + f(2h) + \dots + f((n-1)\cdot h)\right) & \text{if } x = n \cdot h \ \left(\text{for } n \in \mathbb{N}_{n>0}\right) \\ 0 & \text{if } x = 0 \\ \text{undefined otherwise} \end{cases}$$

$Df(x)$ is a continuous function over all $x \in (-\infty, \infty)$

$Sf(x)$ is only defined for $x = nh$ $\underbrace{(n \in \mathbb{N})}_{\{0,1,2,3,4,\dots\}}$

So when $x = nh$

$$DSf(x) = \frac{Sf(\overbrace{x+h}^{(n+1)\cdot h}) - Sf(\overbrace{x}^{nh})}{h}$$

$$= \frac{\cancel{h}\cdot\left(\cancel{f(0)} + \cancel{f(h)} + \dots f(nh)\right) - \cancel{h}\cdot\left(\cancel{f(0)} + \dots + \cancel{f((n-1)\cdot h)}\right)}{\cancel{h}}$$

$$= f(nh) = f(x)$$

and

$$SDf(x) = h\cdot\left(Df(0) + Df(h) + \dots + Df((n-1)\cdot h)\right)$$

$$= \cancel{h}\cdot\left(\frac{\cancel{f(0+h)} - f(0)}{\cancel{h}} + \frac{\cancel{f(2h)} - \cancel{f(h)}}{\cancel{h}} + \frac{\cancel{f(3h)} - \cancel{f(2h)}}{\cancel{h}} + \dots + \frac{f(x) - \cancel{f((n-1)h)}}{\cancel{h}}\right)$$

$$= f(x) - f(0)$$

(h-) Fundamental theorem of calculus

When $x = nh$,

$$SDf(x) = f(x) - f(0)$$

$$DSf(x) = f(x)$$

- From a function f,

  we can extract its rate of change Df

  and DDf, $DDDf$, ... $D^n f$, ...

- From the rate of change of a function, Df

  we can extract the (change of the) function SDf

  and since $\boxed{SDf(x) = f(x) - f(0)}$

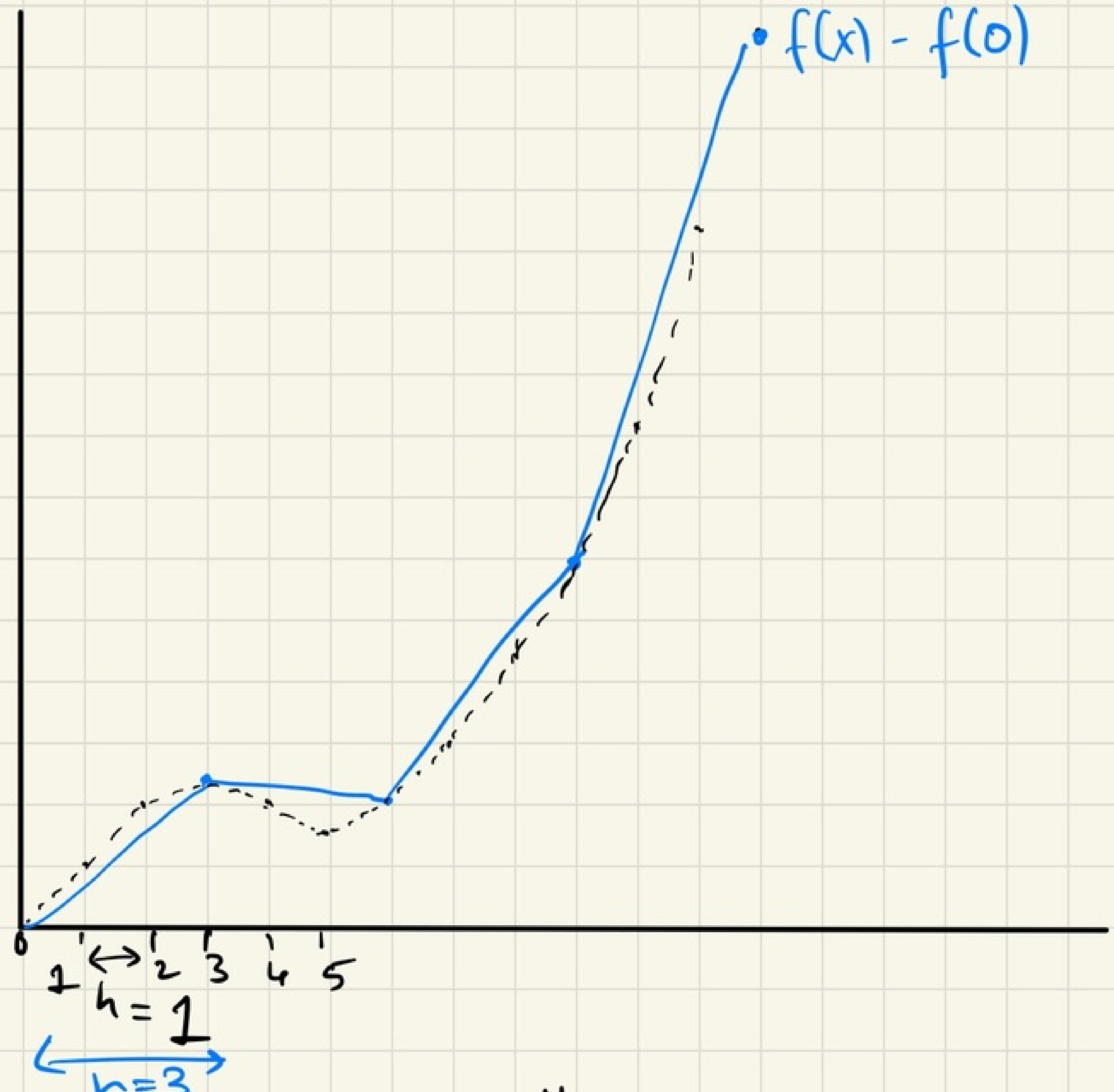
  this is as good as knowing the function up to a constant. (The function's value at 0)

The h-FTC tells us that

if we know Df,

we know $f(x)$ up to a constant (i.e. $f(0)$)

at the points $x = nh$ (multiples of h)



As we make h smaller, we know more points

As $\underbrace{h \longrightarrow 0}$, we know f (the true FTC)

Homo erectus $\rightarrow$ Homo sapiens $\rightarrow$ Homo calculus

The FTC is one of the most important leaps of intellectual evolution in our recent history.

$$\text{Velocity} = D(\underbrace{\text{distance}}_{\text{as a function of time}})$$

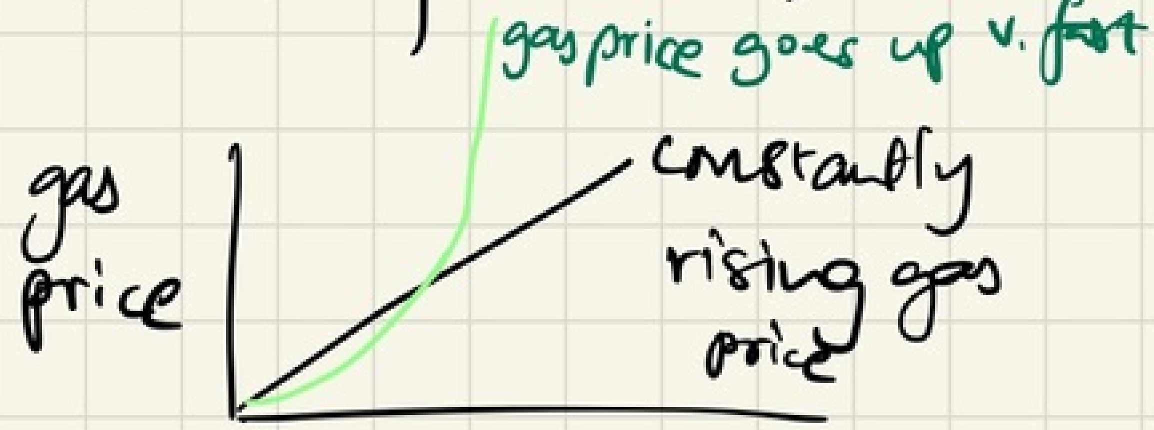
$$\text{Acceleration} = D(\text{velocity}) = DD(\text{distance})$$

$$\text{Force} \approx \text{acceleration} \ (\text{Newton})$$

So if we know/control the force on an object, we know/control where it will be at any future time.

We shoot humans into space in metal tubes at regular intervals

$$\text{inflation} = D(\underbrace{\text{cost of bread/gas}}_{\text{as a function of time}})$$



$$\text{Inflation} > D(\text{wages})$$ "wages rise but people get poorer"

$\rightsquigarrow$ socioeconomic Crisis

Year	Venezuela inflation
2005	15 %
2010	28 %
2015	181 %
2020	2900 %

$$D(\text{population}) = D(\text{births} - \text{deaths})$$
$$= D(\text{births}) - \underbrace{D(\text{deaths})}_{\text{small due to medical advances}}$$

How to Control population despite medical advances / economic growth $\Rightarrow$ One Child policy

$$\text{Power} = D(\underbrace{\text{energy}}_{\text{as a function of time}})$$

$$\text{Electric energy} = \text{voltage} \times \text{charge}$$

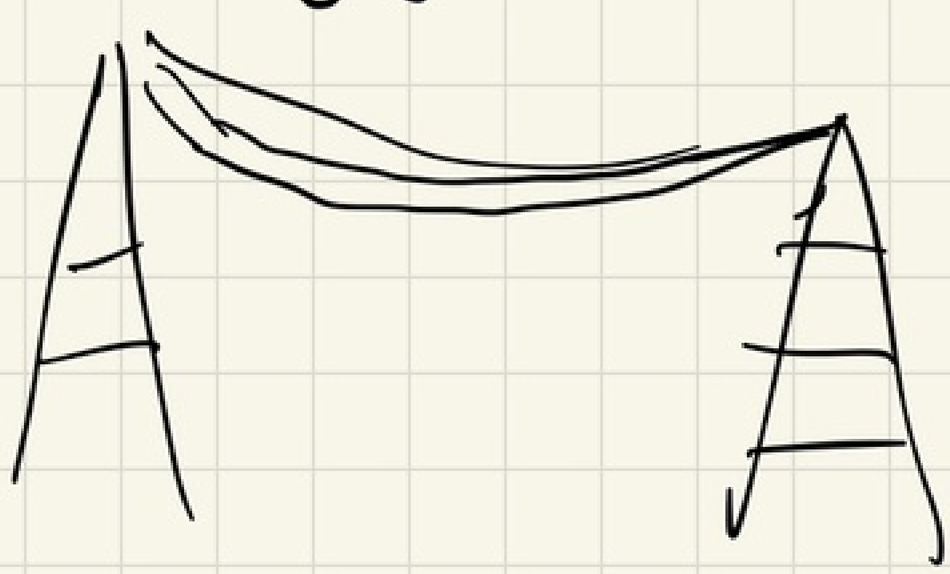
At constant voltage,

$$\text{Power} = \text{voltage} \times D(\underbrace{\text{charge}}_{\text{as a f}^{n} \text{ of time}})$$

D(Charge) is commonly known as

<u>electric</u> <u>current</u> (the thing that gives you a shock)

Q: How to transfer large amounts of electric energy at low current?



High voltage, low current ⇒ high power (high rate of energy transfer)

This is why power lines are at high voltage

A law of physics says that we always have

Δ(entropy) ≥ 0

$\Rightarrow$ Energy flows from hot to cold
(heat) temp

(This is intuitively obvious from experience,
e.g. ice melts in water.
But the reason is that Δ(entropy) ≥ 0)

- Also why refrigerators / AC's require energy (electricity) to transfer heat from cold to hot.

Calculo

~~Cogito~~, ergo sum