Math 130 04 – A Survey of Calculus

Take-home test 2

Due: Tuesday, November 22, 2022, 11:59PM (**hard deadline**)

Instructions:

- This test has **four** questions, each worth **five** points. Your goal is to get **16** points in total.
- Any extra points (> 16) will eventually count towards increasing your grade (A→A^+, B^+→A, B^-→B, and so on) at the end of the semester.
- Each question is divided into subquestions. The points that each subquestion is worth are indicated next to it.
- Write your answers on separate sheets of paper.
- Write your name at the top of **each** page you use, and number each page.
- Number your answers correctly.
- **Justify each step in all your answers fully and clearly.** Answers with no explanation (*even if the calculation is correct*) are worth **zero** points. Answers with a full and correct explanation but a calculation error are worth more than 90% of the points.
- You are expected to work on this test **alone**. Plagiarism will be sanctioned with a fail grade.

1. Calculate the derivatives of the following functions.

   (a) $\boxed{\text{1.5 points}}$ $f(x) = \dfrac{7 \cdot (x^2 - 4)^{\frac{1}{2}}}{x - 2}$

   (b) $\boxed{\text{1.5 points}}$ $f(x) = \ln(x^3 + 4)$

   (c) $\boxed{\text{2 points}}$ $f(x) = e^{(e^x)}$

2. Consider the following real function.

$$f(x) = \begin{cases} e^2 \cdot \dfrac{(x^3+1)}{(x^2+1)} & \text{if } x \leq 0 \\ \\ e^{(3x^2+2)} & \text{if } x > 0 \end{cases}$$

(a) $\boxed{\text{1 point}}$ Is f continuous at 0? Justify your answer.

(b) $\boxed{\text{2 points}}$ Is f differentiable at 0? If so, what is $f'(0)$? Justify your answer.

(c) $\boxed{\text{2 points}}$ Does f have a local maximum or a local minimum in the interval $[-1, 1]$? Justify your answer.

3. A bowl of a hot liquid at 200 degrees Fahrenheit is placed in a room whose ambient temperature is maintained constant at 60 degrees Fahrenheit.

The temperature of the liquid after x minutes is $T(x)$ degrees Fahrenheit, where T is the function:

$$T(x) = 60 + (200 - 60)e^{-kx} \qquad (k \text{ is some constant real number}).$$

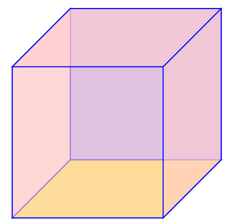
(a) $\boxed{\text{1.5 points}}$ Show that the derivative of T is the function

$$T'(x) = k \cdot (60 - T(x))$$

(b) $\boxed{\text{1.5 points}}$ If after 20 minutes, the temperature of the liquid is 130 degrees Fahrenheit, find the constant k.

(c) $\boxed{\text{1 point}}$ How much time does the liquid take to reach 100 degrees Fahrenheit?

(d) $\boxed{\text{1 point}}$ What are $\lim_{x\to\infty} T(x)$ and $\lim_{x\to\infty} T'(x)$? Try to explain the meaning of this.



4. A juice manufacturer, RainyG, wants to minimize their packaging costs. RainyG sells rectangular cartons of juice of a fixed volume equal to 343 cm^3 (cubic centimeters, or milliliters). The only constraint is that the bottom and top of each carton must be a square (of the same size).

   (a) $\boxed{\text{1.5 points}}$ If the side of the bottom and top square has length x centimeters, what is the height $h(x)$ of the carton? (Hint: the volume of the carton is $h(x) \cdot x^2$ cubic centimeters.)

   (b) $\boxed{\text{1.5 points}}$ Find the surface area of the carton (in square centimeters) if the side of the bottom and top square has length x centimeters. (Hint: The surface area of the carton is the sum of the areas of all its 6 faces (the top, bottom, front, back and the two sides).)

   (c) $\boxed{\text{2 points}}$ The packaging material costs RainyG 0.1 cents per square centimeter. So if the surface area of the carton is $A(x)$ square centimeters (where x is the length of the side of the bottom and top face), then RainyG's cost function is

   $$C(x) = 0.1 \cdot A(x) \qquad \text{cents per carton.}$$

   Does the cost function have a minimum? If so, what is it (in cents per carton)?