Math 130 04 – A Survey of Calculus

Midterm Cheat-sheet

October 18, 2022

Rational functions

Remember: A *rational function* is a real function of the form $f(x) = \dfrac{p(x)}{q(x)}$, where p and q are 1-variable polynomials. The domain of a rational function $f(x) = \dfrac{p(x)}{q(x)}$ is the set of all real numbers *except* for the roots of q (the real numbers a such that $q(a) = 0$).

Algebra of limits

Let f and g be any real functions. If a is any real number or ∞ or $-\infty$, and if the limits $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist, then

- **Sum:** $\lim_{x\to a} (f(x) + g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$
- **Product:** $\lim_{x\to a} f(x) \cdot g(x) = \left(\lim_{x\to a} f(x)\right) \cdot \left(\lim_{x\to a} g(x)\right)$
- **Quotient:** If $\lim_{x\to a} g(x)$ is not equal to 0, then $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$

If $f(x) = \dfrac{p(x)}{q(x)}$ is a rational function, and if a is a root of *both* polynomials p and q, then $\lim_{x\to a} f(x) = \lim_{x\to a} \dfrac{p_1(x)}{q_1(x)}$, where $p(x) = p_1(x) \cdot (x - a)$ and $q(x) = q_1(x) \cdot (x - a)$.

Algebra of continuity

Remember: A real function f is **continuous** at a real number a if $f(a)$ is defined and if $\lim_{x\to a} f(x) = f(a)$.

If f and g are continuous at a, then:

- **Sum:** $(f + g)(x) = f(x) + g(x)$ is continuous at a.
- **Product:** $(f \cdot g)(x) = f(x) \cdot g(x)$ is continuous at a.
- **Quotient:** If $g(a) \neq 0$, then $\dfrac{f}{g}(x) = \dfrac{f(x)}{g(x)}$ is continuous at a.

Derivatives

Remember: Let f be a continuous real function. The *derivative* of f at a real number x is defined to be the limit:

$$f'(x) = \lim_{h\to 0} D_h f(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

If the derivative of f exists at every real number x in an interval I, then f is **differentiable** over the interval I.

Remember: If f is differentiable over I, then the derivative f' is a real function that is *continuous* over I.

Remember: A function f is **strictly increasing** at a real number a if $f'(a) > 0$.

Remember: A function f is **strictly decreasing** at a real number a if $f'(a) < 0$.

Rules for derivatives

Let f, g be real functions that are differentiable over an interval I. Then,

- **Sum rule:**
$$(f+g)'(x) = f'(x) + g'(x)$$
- **Product rule:**
$$(f \cdot g)'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$
- **Quotient rule:** If $g(x)$ is never 0 over I, then
$$\frac{f}{g}(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$$

Some useful derivatives

The derivatives of some useful functions are given below.

- If $f(x) = a$ (for some constant real number a), then $f'(x) = 0$.
- If $f(x) = x^a$ (for some constant real number a), then $f'(x) = a \cdot x^{a-1}$.
- If $f(x) = a \cdot g(x)$ (for some constant real number a and some function g), then $f'(x) = a \cdot g'(x)$.