Math 150 03 – Calculus I

Homework assignment 5

Due: Wednesday, November 1, 2023

1. (a) Find $\frac{d}{dx}[y]$ in terms of x and y if we have that $x \cdot \ln(y) + y^3 = 3 \cdot \ln(x)$.
   (b) Use implicit differentiation to find the tangent line to the curve $x = y^5 - 5y^3 + 4y$ at the point $(0, 1)$.
   (c) Use implicit differentiation to find the tangent line to the curve $\sin(x + y) + \cos(x - y) = 1$ at the point $(\frac{\pi}{2}, \frac{\pi}{2})$.
2. Use L'Hôpital's rule where appropriate to find the following limits.
   (a) $\lim_{x \to 4} \dfrac{\ln(\frac{x}{4})}{x^2 - 16}$
   (b) $\lim_{x \to 0} \dfrac{1 - \cos(7x)}{1 - \cos(3x)}$
   (c) $\lim_{x \to 1} \dfrac{4^x - 3^x - 1}{x^2 - 1}$

   (L'Hôpital's rule at $\pm\infty$)
   When $x \to a$ (where a is any real number or $\pm\infty$), L'Hôpital's rule states that if $f(x)$ and $g(x)$ both approach 0 or both approach $\pm\infty$, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

   provided the right hand limit exists, and provided $g'(x) \neq 0$ whenever $x \in (a - h, a) \cup (a, a + h)$ for some $h > 0$ (or, if $a = \pm\infty$, whenever $x \in (h, \infty)$ or $(-\infty, h)$ as the case may be).
3. Evaluate the following limits, using L'Hôpital's rule as appropriate.
   (a) $\lim_{x \to \infty} \dfrac{15x^3}{e^{2x}}$
   (b) $\lim_{x \to \infty} \dfrac{e^x + x}{e^x + x^2}$
4. We say that a function g **dominates** a function f when we have $\lim_{x \to \infty} f(x) = \infty$, $\lim_{x \to \infty} g(x) = \infty$, and $\lim_{x \to \infty} \dfrac{f(x)}{g(x)} = 0$.
   (a) Which function dominates the other: $\ln(x)$ or $\sqrt{x}$?
   (b) Which function dominates the other: $\ln(x)$ or $x^{1/n}$? (n is any natural number bigger than 1)
   (c) Explain why e^x dominates every polynomial.