Math 150 03 – Calculus I

Homework assignment 7

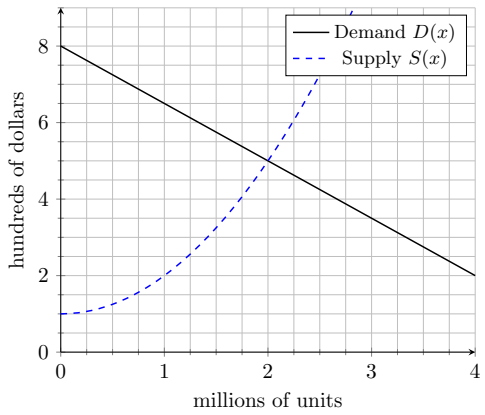
Due: Wednesday, November 15, 2023

Recall the useful form of the Fundamental Theorem of Calculus:

FTC(2): If $\left(\int f\right)$ is any antiderivative of f, then we have that

$$\int_a^b f(x) \cdot dx = \left(\int f\right)(b) - \left(\int f\right)(a)$$

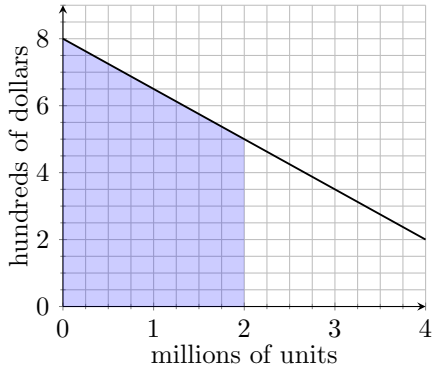
1. Calculate the following definite integrals by first finding an antiderivative of the given function and then using the FTC(2).
   (a) $\displaystyle\int_1^2 [x \cdot \ln(x)] \cdot dx$
   (b) $\displaystyle\int_0^1 \left[e^{-x} \cdot (2e^{-x} + 3)^9\right] \cdot dx$
   (c) $\displaystyle\int_0^{\frac{\pi}{4}} \left[x \cdot (\sec(x))^2\right] \cdot dx$
2. For each pair of curves given below, find two points where they intersect and calculate the area of the region bounded by them. (Hint: you can use a graphing calculator to visualise the curves, but you should find their points of intersection using a calculation.)
   (a) $f(x) = \sqrt{x}$ and $f(x) = \frac{1}{4}x$.
   (b) $f(x) = 12 - 2x^2$ and $f(x) = x^2$.
3. (Supply and demand)



In economics, the *demand curve* of a product is a function $D(x)$ where x is the total number of products that consumers are willing to buy at a price point of $D(x)$ dollars per unit of that product. In general, D is a decreasing function (since consumers will usually buy more of a product only if the price per unit is less).

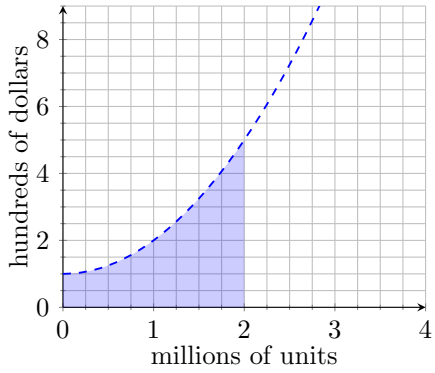
Conversely, the *supply curve* of a product is a function $S(x)$ where x is the total number of products that suppliers are willing to sell at a price point of $S(x)$ dollars per unit of that product. In general, S is an increasing function (since suppliers will usually supply more of a product only if the price per unit is higher).

(a) The demand curve for espresso machines is given by $D(x) = 8 - \frac{3}{2}x$ hundred dollars per unit, assuming x million units are bought by consumers.



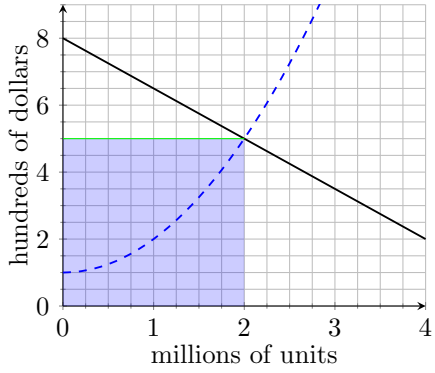
Explain why the area of the shaded region is the *maximum* total amount of money that consumers are willing to spend to buy 2 million espresso machines. What is this amount?

(b) The supply curve for espresso machines is given by $S(x) = 1+x^2$ hundred dollars per unit, assuming x million units are sold by suppliers.



Explain why the area of the shaded region is the *minimum* total amount of money that suppliers are willing to sell 2 million espresso machines for. What is this amount?

(c) The point where the supply and demand curves of a product intersect is called the *market equilibrium*. The x-coordinate of this point represents the number of units that are bought and sold in reality, and the y-coordinate is the actual price at which each unit is sold for.



i. Use the figure to find the market equilibrium for espresso machines. How many espresso machines are bought and sold in reality, and what is the final price per unit of each one?

ii. Explain why the rectangular shaded region in the figure represents the total amount of money given by consumers to suppliers if the number of espresso machines bought and sold, as well as the price of each espresso machine, are given by the market equilibrium.

iii. The difference between the area of the rectangular region and the area calculated in question (a) is called the *consumer surplus*. Calculate this amount for espresso machines. Explain why this is the amount of money that consumers *save* if the number of espresso machines bought and sold, as well as the price of each machine, is determined by the market equilibrium.

iv. The difference between the area of the rectangular region and the area calculated in question (b) is called the *supplier surplus*. Calculate this amount for espresso machines. Explain why this is the amount of money that suppliers *gain* if the number of espresso machines bought and sold, as well as the price of each machine, is determined by the market equilibrium.

(d) Explain why a fixed price per unit (determined by the market equilibrium) is good for both consumers and suppliers (as opposed to the situations in questions (a) and (b)).