Math 150 03 – Calculus I

Homework assignment 8

Due: Monday, December 11, 2023

1. **Reversing the bounds of integration:** If $a \leq b$ are two real numbers, and if f is a real function that is continuous over the interval $[a, b]$, then the definite integral $\int_b^a f(t) \cdot dt$ is defined as:

$$\int_b^a f(t) \cdot dt = -\int_a^b f(t) \cdot dt$$

Consider the following definition.

$$f(x) = \begin{cases} x^5 + 2x^2 - 2 & \text{if } x \leq 0 \\ \\ x^3 + 4x - 2 & \text{if } x > 0 \end{cases}$$

   (a) Is the function f continuous over $(-\infty, \infty)$? Namely, is f continuous at every real number x?
   (b) If $x \leq 0$, what is the value of $\int_0^x f(t) \cdot dt$?
   (c) If $x > 0$, what is the value of $\int_0^x f(t) \cdot dt$?
   (d) Write a piecewise definition of an antiderivative of f.

2. The *marginal cost function* is the derivative of the total cost function. Similarly, the marginal revenue and marginal profit functions are the derivatives of the total revenue and total profit functions respectively.

   A company calculates its marginal cost function C' as follows: If x thousand units have been produced, the marginal cost (i.e. the cost to produce the next unit) is

$$C'(x) = 2x^{-1/3} \qquad \text{dollars per unit.}$$

   (a) Explain why the units of the marginal cost function C' are (dollars per unit), while the units of the cost function C are (thousands of dollars).
   (b) Find the company's total cost function C (i.e. $C(x)$ thousands of dollars to produce x thousand units) if the fixed cost to produce 0 units is 3 thousand dollars (i.e. $C(0) = 3$).
   (c) Suppose the company's marginal revenue function is as follows: If x thousand units have been sold, the marginal revenue (i.e. the revenue from selling the next unit) is

$$R'(x) = 3x^{-1/2} \qquad \text{dollars per unit.}$$

      i. Find the marginal profit function P'. (Remember that the total profit function P is defined as $P(x) = R(x) - C(x)$, where R and C are the total revenue and total cost functions.)
      ii. Does the total profit function P have a maximum in the interval $[0, 20]$? If so, find the value $a \in [0, 20]$ such that P has a maximum at a.
      iii. Calculate the total profit function P, assuming that the revenue from selling 0 units is 0 dollars (i.e. $R(0) = 0$).