Math 130 04 – A Survey of Calculus

Homework assignment 3

Due: Tuesday, September 20, 2022

Instructions: Write your answers on a separate sheet of paper. Write your name at the top of each page you use, and number each page. Number your answers correctly.
Justify all your answers.

1. Consider the following definition.

$$f(x) = \begin{cases} \frac{1}{x^2+2x} & \text{if } x < 0 \\ 0 & \text{if } x > 0 \end{cases}$$

   (a) Does this define a real function? If so, what is its domain? Justify your answer.

   (b) Do the following limits exist? If so, what are they? Justify your answers.

      i. $\lim_{x \to -2^-} f(x)$ (the left-hand limit of f at -2.)

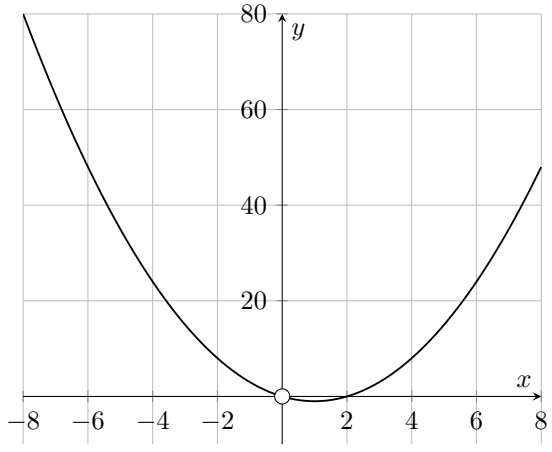
      ii. $\lim_{x \to -1} f(x)$ (the limit of f at -1.)

      iii. $\lim_{x \to 0^+} f(x)$ (the right-hand limit of f at 0.)

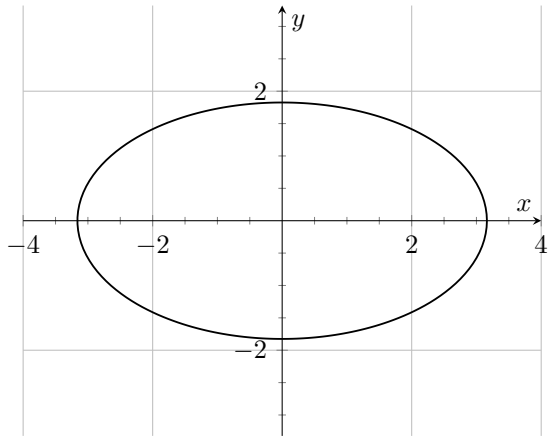
      iv. $\lim_{x \to 0} f(x)$

2. For each of the following graphs, justify whether the limit at $x = 0$ exists.

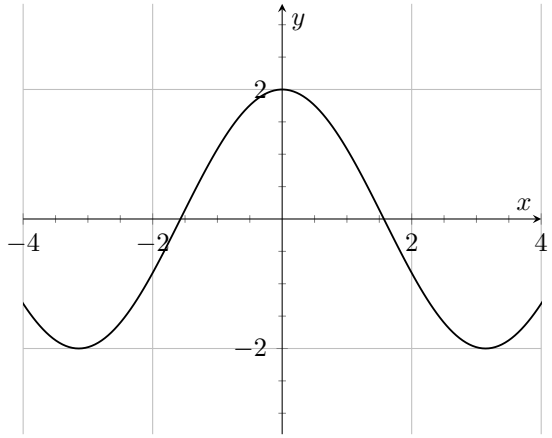
   (a)



(b)



(c)



3. Find the following limits. Justify your answers.
   (a) $\lim_{x \to 0} x^2 + 4$
   (b) $\lim_{x \to \infty} \frac{x^2}{x^2 - 1}$
   (c) $\lim_{x \to 1} \frac{x^2 + 3x - 4}{x^2 - 1}$