Math 130 04 – A Survey of Calculus

Midterm Exam

October 18, 2022; **Time: 1 hour+**

Your Name:

Your Student ID:

Instructions:

- This exam has **five** questions, each worth **five** points. Your goal is to get **18** points in total.
- You have 1 hour (and a bit) to finish the exam.
- Any extra points (> 18) will eventually count towards increasing your grade (A$\to$A$^+$, B$^+\to$A, B$^-\to$B, and so on) at the end of the semester.
- Each question is divided into subquestions. The points that each subquestion is worth are indicated next to it.
- Write your answers clearly and neatly in the space provided after each question.
- Ask for extra sheets of paper if you need them.
- Number your answers correctly (especially if you're using extra sheets of paper).
- Justify your answers **fully and clearly.** Answers with no explanation (*even if the final calculation is correct*) are worth **zero** points. Answers with a full and correct explanation but a calculation error are worth more than 90% of the points.

1. Consider the following definition.

$$f(x) = \begin{cases} \dfrac{3x^3 - 7x^2 + 10}{2x + 2} & \text{if } x < 0 \\ 4x^2 + 5 & \text{if } x \geq 0 \end{cases}$$

(a) (1½ points) Is f a real function? If so, what is its domain?

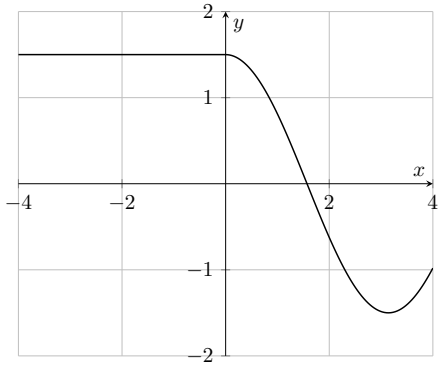
(b) (1½ points) Is f continuous at 0?

(c) (1 point) Is f differentiable over the interval $[1, 3]$?

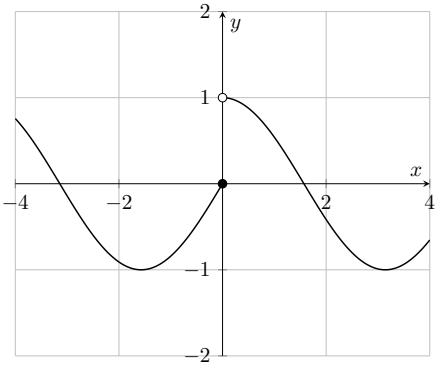
(d) (1 point) What is $\lim_{x \to -1} f(x)$?

2. (a) (2 points) Which of the following graphs represent real functions? Which of the functions is continuous over the interval $[-1, 1]$?

i.



ii.

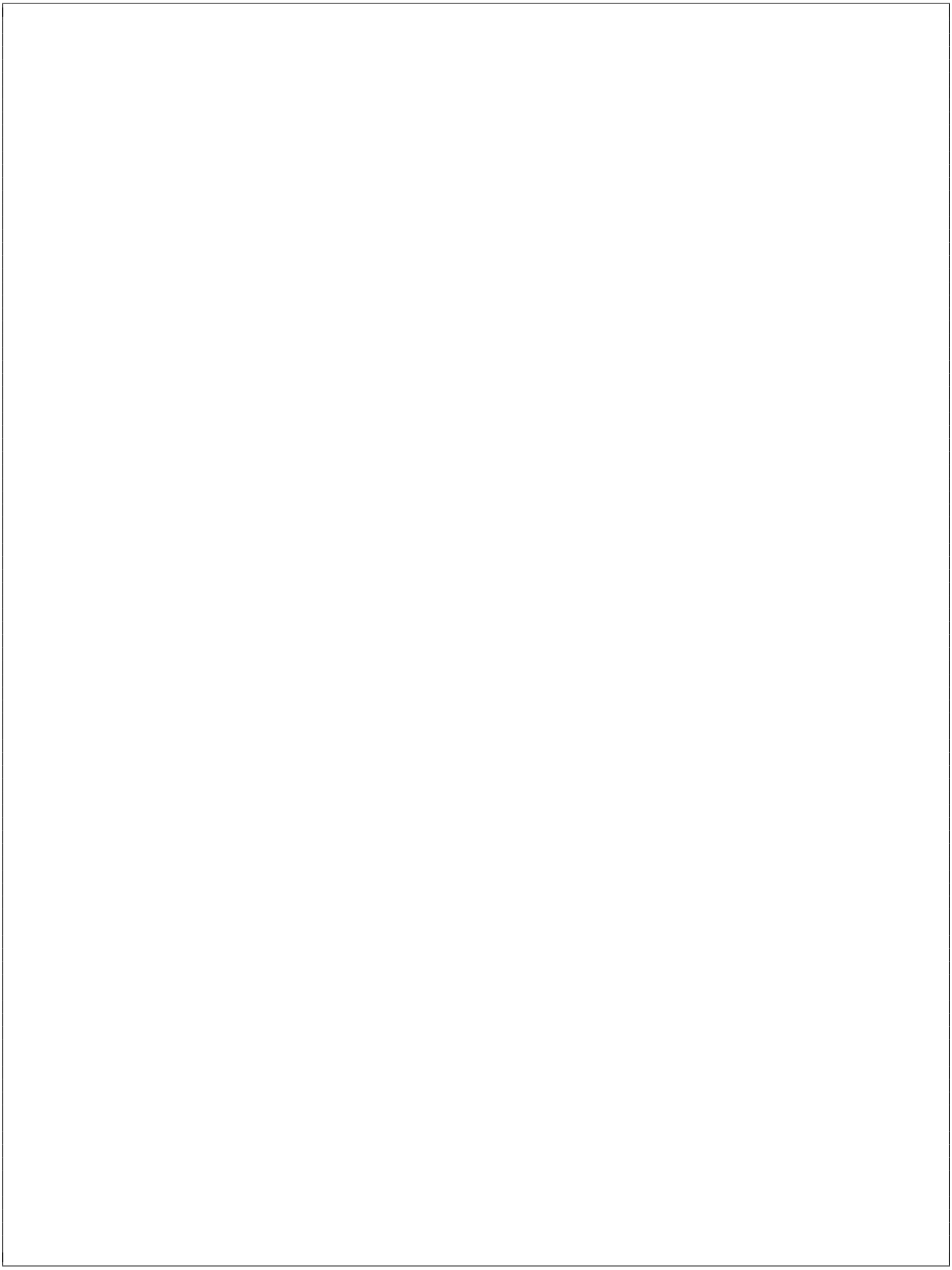


(b) (3 points) Calculate the following limits.

i. $\lim_{x\to\infty} \dfrac{3x^3 - 13x}{x^3 + 3}$

ii. $\lim_{x\to 2} \dfrac{4x^3 - 5x^2 - 2x - 8}{x^2 - 4}$

iii. $\lim_{x\to-\infty} \dfrac{3x^3 - 13x}{x^3 + 3}$

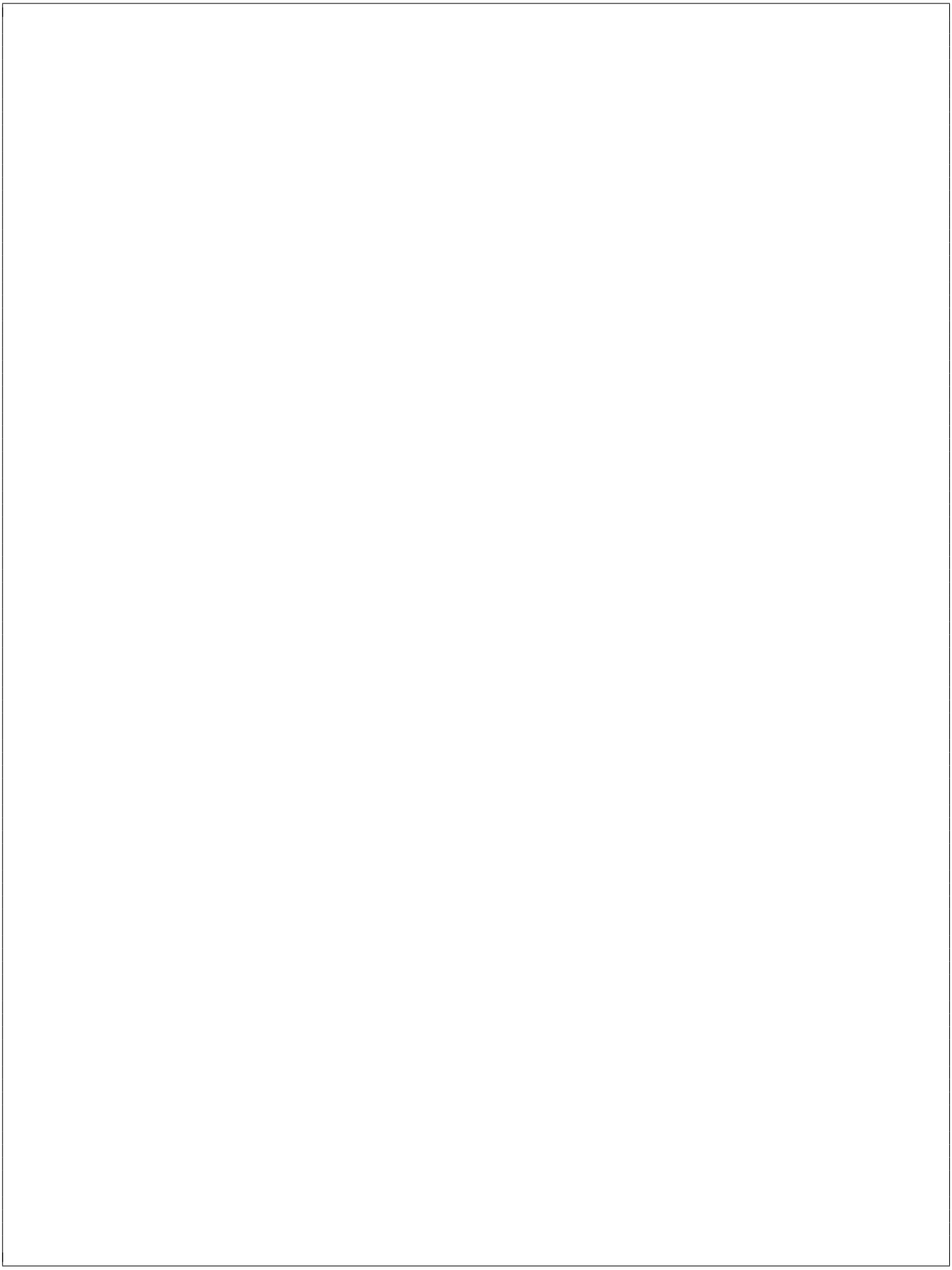


3. Calculate the derivatives of the following functions.

(a) (1½ points) $f(x) = \dfrac{13x^3 - 7x}{2x + 3}$

(b) (1½ points) $f(x) = 7 + 2x^{\frac{1}{3}}$

(c) (2 points) $f(x) = \left(\dfrac{7}{x^{\frac{1}{4}}} + 8x^3\right) \cdot \left(x^2 - 1\right)$



4. A juice manufacturing company, RainyG, calculates its cost of producing x cartons of apple juice to be $C(x)$ dollars, where $C(x) = 40 + 1.5x^{1/2}$. RainyG also estimates its revenue from selling x cartons of apple juice to be $R(x)$ dollars, where $R(x) = 6x^{1/2}$.

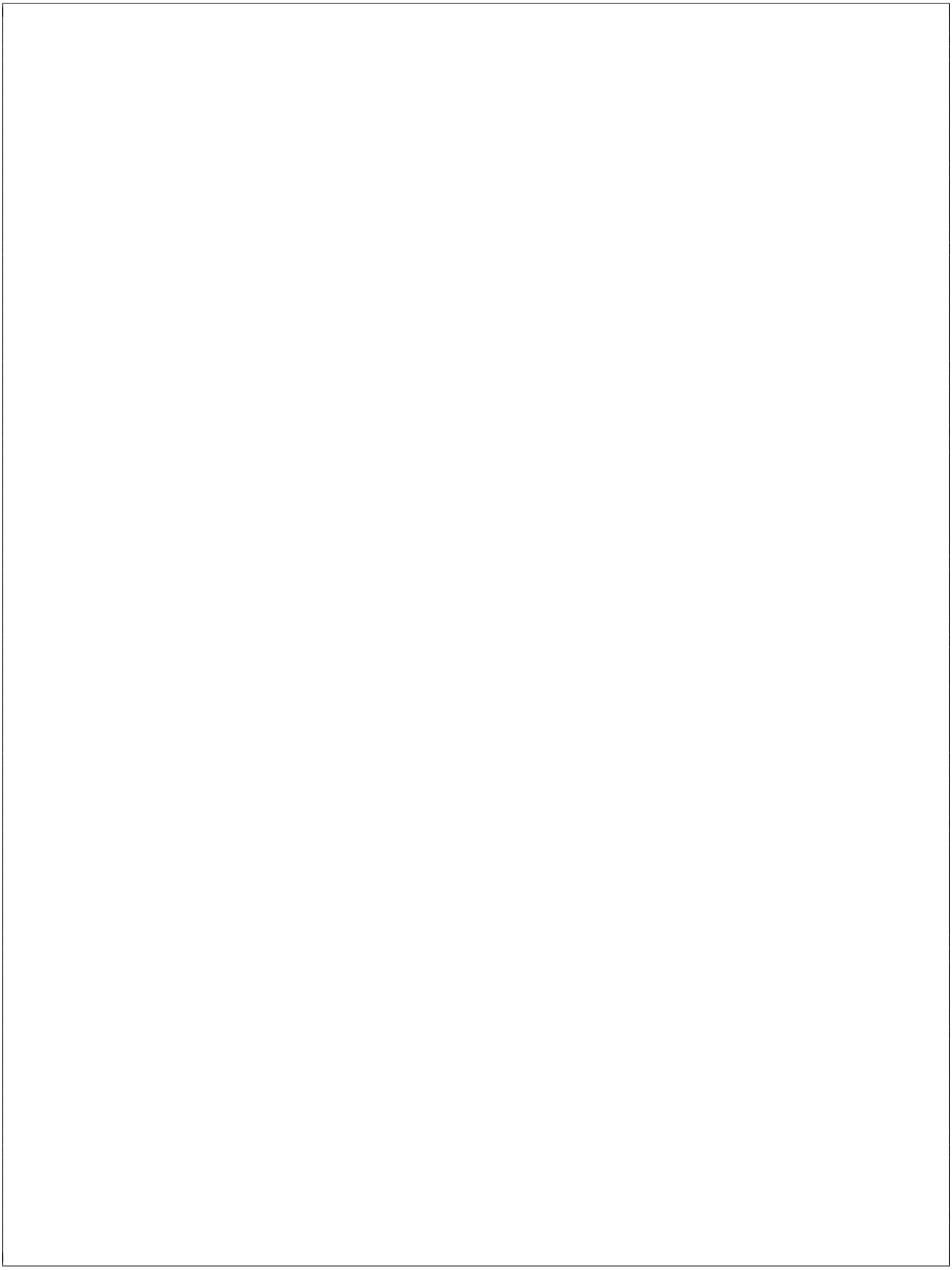
   (a) (1 point) What is RainyG's average cost per carton from producing and selling 100 cartons of apple juice?

   (b) (1 point) What is RainyG's average revenue per carton from producing and selling 100 cartons of apple juice?

   (c) (1 point) What is RainyG's average profit per carton from producing and selling 100 cartons of apple juice?

   (d) (1 point) What is the rate of change of RainyG's average profit per carton when 100 cartons are produced and sold? Is the average profit increasing or decreasing?

   (e) (1 point) What is the rate of change of RainyG's average profit per carton when 400 cartons are produced and sold? Is the average profit increasing or decreasing?



5. A study shows that the resting heart rate (in beats per minute, or bpm) of an active person who is $10 \cdot x$ years old can be modeled by the function:

$$f(x) = \frac{x^3}{100} - \frac{x^2}{2} + 4x + 55$$

Therefore at the age of x decades ($10 \cdot x$ years), the person has a resting heart rate of $f(x)$ beats per minute.

(a) (1 point) What is an active 20-year-old person's resting heart rate (in bpm)?

(b) (1 point) What is the rate of change of an active 20-year-old's resting heart rate (in bpm per decade)?

(c) ($1\frac{1}{2}$ points) Calculate $f'(4)$. Will the resting heart rate of an active 40-year-old increase or decrease as they age?

(d) ($1\frac{1}{2}$ points) Calculate $f'(5)$. Will the resting heart rate of an active 50-year-old increase or decrease as they age?