Math 150 03 – Calculus I

Homework assignment 2

Due: Wednesday, September 20, 2023

Instructions: Write your answers on a separate sheet of paper. Write your name at the top of each page you use, and number each page. Number your answers correctly. **Justify all your answers.**

1. Consider the following definition. (Recall that $|x|$ is the absolute value of x.)

$$f(x) = \frac{|x|}{x}$$

   (a) Does this define a real function? If so, what is its domain? Justify your answer.

   (b) Draw the graph of f.

   (c) Do the following limits exist? If so, what are they? Justify your answers.

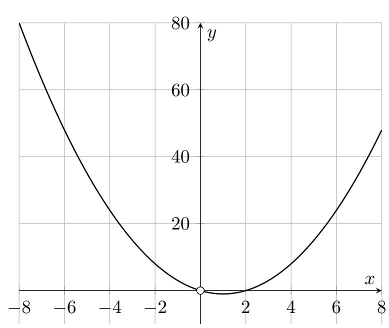
      i. $\lim_{x \to -2^-} f(x)$ (the left-hand limit of f at -2.)

      ii. $\lim_{x \to 0^+} f(x)$ (the right-hand limit of f at 0.)

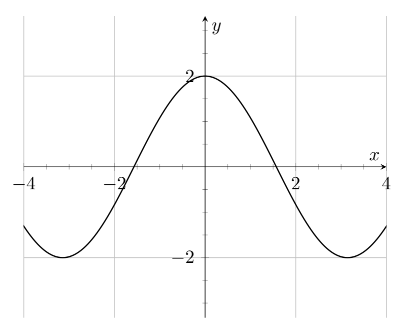
      iii. $\lim_{x \to 0} f(x)$

2. Which of the following are functions? What are their limits at $x = 0$? Are they continuous at $x = 0$?

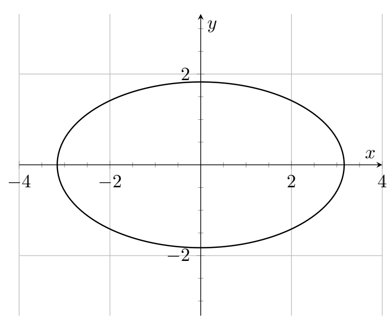
(a)



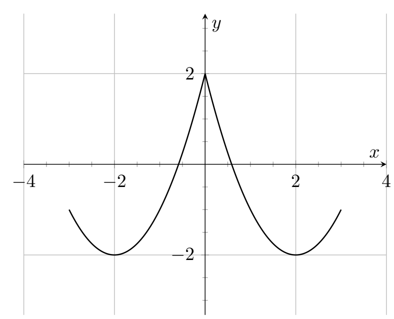
(c)



(b)



(d)



3. Find the following limits. Justify your answers. (Hint: you can use a graphing calculator like **GeoGebra** to *see* what the functions look like, but you should give a complete justification of your answer.)

(a) $\lim_{x\to 0} x^2 + 4$ (b) $\lim_{x\to\infty} \frac{x^2}{x^2-1}$ (c) $\lim_{x\to 1} \frac{x^2+3x-4}{x^2-1}$

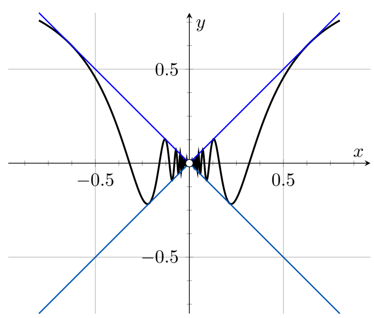
4. **(The squeeze theorem.)**

Theorem (Squeeze Theorem)**.** *Let $f, g, h\colon A \to \mathbb{R}$ be real functions such that for every $x \in A$, we have the inequalities $g(x) \leq f(x) \leq h(x)$. If $a, b \in \mathbb{R}$ are real numbers such that*

$$\lim_{x\to b} g(x) = \lim_{x\to b} h(x) = a,$$

then we have that $\lim_{x\to b} f(x) = a$.

We would like to use the squeeze theorem to show that $\lim_{x\to 0} \left(x \cdot \sin \frac{1}{x}\right) = 0$.



(a) What is the domain of the function $f(x) = x \cdot \sin \frac{1}{x}$?

(b) Show that, for any x in the domain of f, we have the inequalities $-|x| \leq x \cdot \sin \frac{1}{x} \leq |x|$.

(c) Use the graph of the absolute value function to show that $\lim_{x\to 0} |x| = 0$.

(d) Use the squeeze theorem to show that $\lim_{x\to 0} \left(x \cdot \sin \frac{1}{x}\right) = 0$.