Math 150 03 – Calculus I

In-class problem sheet

Thursday, October 4, 2023

1. Evaluate the derivatives of the following functions (over any interval where they are defined).

   (a) $f(x) = x^3 - 9x^2 + 16$

   (b) $f(x) = \dfrac{x^3 + 25}{3x - 2}$

   (c) $f(x) = \dfrac{x + 3}{x^2 - 4}$

   Remember: The **chain rule** says that if f and g are real functions, then the derivative of the composite function $(g \circ f)(x) = g(f(x))$ can be calculated as follows.

   $$(g \circ f)'(x) = g'(f(x)) \cdot f'(x)$$

2. Evaluate the derivatives of the following functions.

   (a) $f(x) = (x^2 + 3x + 4)^3$

   (b) $f(x) = (x^3 + 5)^{1/4}$

   (c) $f(x) = \dfrac{x + 3}{(x^2 - 4)^{2/3}}$

3. The distance between Los Angeles and San Diego on the I-5 highway is 118 miles.

   (a) What is the average speed (in mph) required to do the trip in 1.5 hours?

   (b) If the speed limit is 70 mph all along the I-5, is it possible to do the trip in 1.5 hours without breaking the law? Explain. What about in 2 hours?

   (c) A car going from L.A. to San Diego on the I-5 travels $f(x)$ miles (measured from L.A.) after x hours, where f is the function:

   $$f(x) = x(91 - 16x)$$

      i. How long does the car take to reach San Diego (i.e. cover 118 miles)?

      ii. Does the car ever break the law? (Is its speed ever more than 70 mph?)

      iii. What is the car's speed when it leaves L.A. (at the starting time)? What is the car's speed when it arrives in San Diego? (That is, at the time calculated in part i.)

   Remember: Speed (or velocity) is the derivative of distance as a function of time.