Math 150: Calculus I

Lecture -1: Where it all started

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Introduction

Why calculus is useful to you

The historical origins of calculus

Before we begin…

General info/rules:

All course material will be available on the website: https://www.chaitanyals.site/teaching/math150

Each of you should have a copy of the syllabus. Read it carefully.

You can call me "Chaitanya", "Professor", or any combination of "Professor/Mr./Dr." followed by "Chaitanya/S./L.S./Subramaniam/Leena Subramaniam".
Basically, call me anything except "hey, you!"

You can enter and leave class at any point without asking—just do so quietly.

Before we begin...

General suggestions:

The quality of the notes you take in class will make a massive difference to your understanding. And your grade. So take notes.

Re-read your notes after class. If you don't understand something, *come in to office hours*.

If you miss class, come in to office hours.

If you don't understand the homework, come in to office hours.

Come in to office hours.

If you reallly don't want to come in to office hours, the Math Learning Center is a great resource!

Brass tacks

Why learn calculus?

Conceptual reasons: it's used to model/study tons of stuff: the natural sciences (physics, chem, bio), statistics, economics, business, engineering, music, medicine, computer science, telecommunications etc. etc.

To do more (modern) math: High school math *is nothing like* the way we do math today. A solid understanding of how calculus works is a necessary step to get through the inflection point.

Philosophical/historical reasons: Calculus is a pretty simple idea, yet it completely revolutionized the way we think. Why?

Practical reasons: To get an A in this class.

Introduction

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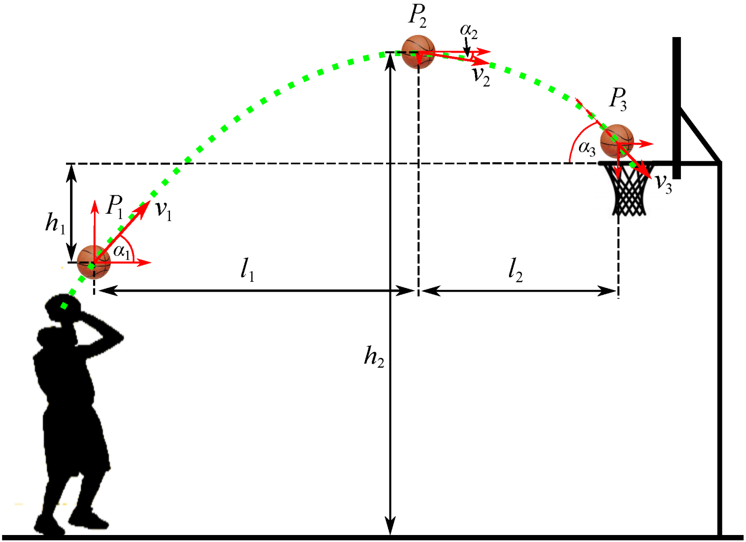
Physics and engineering

What is the angle and speed at which a player of a given height should shoot in order to score from a given distance? Can this be optimised?



Economics

A company can sell $y(x)$ million units at a price of x hundred dollars per unit. Given that (in economies of scale) producing more units *decreases* the production cost per unit, what is the number of units the company should produce and sell to maximize profit?

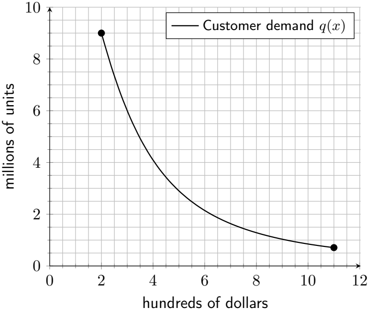


Figure: Market demand model

Probability

Studies show that most probable time that a random student takes to finish a 60 minute test is 48 minutes.

If a million (identical, random) students take the same test, their average finishing time is:

(a) About 48 minutes.

(b) I'll get back to you after finishing Calculus I.

A warning about history

Baez's Law

"Any effect, constant, theorem or equation named after Professor X was first discovered by Professor Y, for some value of Y not equal to X."

Of course, Baez's Law applies to itself.

Renaissance algebra

By the 1500s, the Italians had figured out formulas to solve cubic and quartic equations by hand (published in Cardano's *Ars Magna*).

$$ax^3 + bx^2 + cx + d = 0 \qquad ax^4 + bx^3 + cx^2 + dx + e = 0$$

Some of these were quite complicated compared to the quadratic formula known to antiquity (and hopefully to you).

Recap: Quadratic formula

The solutions to the quadratic equation $ax^2 + bx + c = 0$ are

$$r_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \qquad r_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Takeaway

Math before calculus was a bag of increasingly complicated tricks.

The history of the people behind the cubic/quartic solutions is interesting.

(l to r) Tartaglia, Cardano and Ferrari.

Math in Italy at the time was rough—there was no tenure!

So people kept their work secret and were furious when it was “stolen”.

Cardano got Tartaglia to send him (Cardano) his (Tartaglia’s) cubic solution by promising not to publish it. Then he (Cardano) published it.

Except Ferrari (Cardano’s student/servant) actually did most of the work in *Ars Magna*. Ferrari was later murdered (possibly by his own sister).

Analytic Geometry (Fermat and Descartes)

The tedious mathematics done by colorful mathematicians of the 1500s gave way in the 1600s to dreary mathematicians (Pierre Fermat's day job was being a mediocre lawyer in Toulouse)...who did *incredibly* creative mathematics.

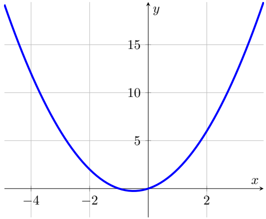
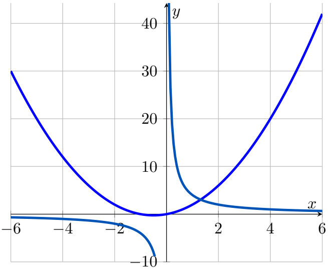


Figure: The points (x, y) that are solutions of $x^2 + x = y$ trace a parabola.

Fermat realized that solutions to quadratic equations in *two* variables were certain curves in the plane (called conic sections—a.k.a. hyperbolas, parabolas and ellipses).

Fermat then reinterpreted the Italians' solutions to cubic (and more general) equations as certain points on these curves. For example, starting with the cubic equation $x^3 + x^2 = 4$,

> Set both sides equal to xy and simplify to get two quadratic equations in two variables: $x^2 + x = y$ and $4 = xy$.
>
> Each of these is now a curve. If the curves intersect at some point (a, b) in the plane, then a is a solution of the original *cubic* equation!



Fermat's "Adequality" and maximization problems

From studying curves, Fermat figured out the following algorithm to find the maximum value of a polynomial (say, $p(x) = 3x - x^2$).

Suppose x and $x + e$ are two arbitrary roots, we can write the "adequation" $p(x) \approx p(x + e)$. In our example, we get $3x - x^2 \approx 3x + 3e - x^2 - 2ex - e^2$.

Cancelling common terms, we get the adequation $3e \approx 2ex + e^2$.

Dividing through by e, we get $3 \approx 2x + e$.

Ignoring multiples of e, we get that $p(x)$ has a maximum at $x = \frac{3}{2}$.

This is the exact same algorithm that we will justify using differential calculus! (Fermat figured it out intuitively using the geometry of curves.)

Descartes' *Géométrie*

Fermat generalized the maximization problem to find the slope of the tangent line to any point on a curve (essentially, the value of the derivative at that point).

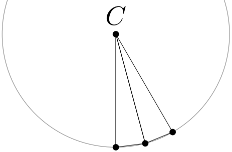
While Fermat's ideas were revolutionary, his notation was bad, his statements weren't super general, and he didn't really give many proofs.

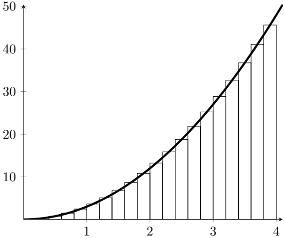
At the same time, the philosopher and polymath René Descartes had many of the same ideas about curves, tangents and polynomial equations.

His presentation of these ideas in *La Géométrie* was hard to read, but extensive and systematic. Descartes treated more complicated examples than Fermat, his notation was also mostly the same as what we use today, and he wrote in French (Fermat wrote in Latin).

The area under curves

Fermat (and Roberval) also found (reinvented?) a way to calculate areas under polynomials, following ideas of Kepler, Cavalieri, Galileo and others.





So did Fermat actually invent calculus?

Fermat did not realize the inverse relationship between the derivative and the integral (a.k.a. "the fundamental theorem of calculus")

Fermat had no concept of *function*, which is the abstract mathematical concept needed to make the required leap of imagination.

It would take Newton and Leibniz to invent calculus—however even they didn't actually explicitly work with functions!

TBC. . .

Tomorrow

You have a quiz in tomorrow's class, so don't be late! (No need to prepare.)