Math 150 03 – Calculus I

Practice Exam

December 13, 2023 **Time: 2 hours**

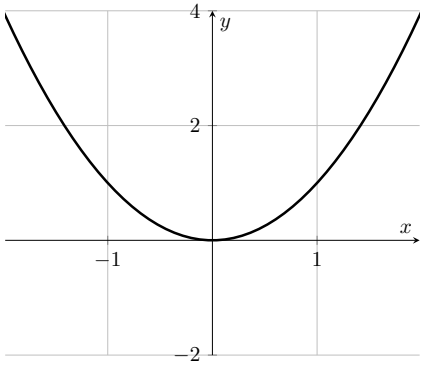
Instructions:

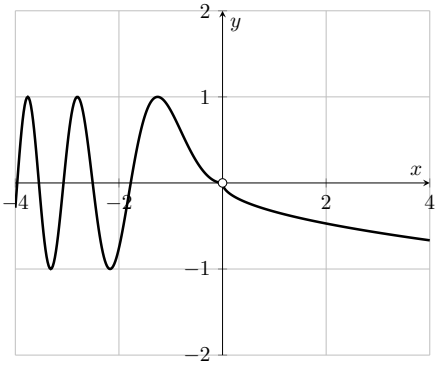
- You have exactly 2 hours to finish the exam.
- You are allowed to use your personal notes (paper only) and a scientific (non-graphing) calculator. No other devices (computers, cell phones, tablets) may be used.
- You **must** write your name and student ID at the top of the first page, and you **must** initial every page that you use.
- This exam has questions worth **20** points in total. In order to score 100%, you need to get **16** points in total.
- Any extra points (> 16) will eventually count towards increasing your grade (A$\to$A$^+$, B$^+\to$A, B$^-\to$B, and so on) at the end of the semester.
- Each question is divided into subquestions. The points that each subquestion is worth are indicated next to it.
- Write your answers clearly and neatly in the space provided after each question.
- Ask for extra sheets of paper if you need them.
- Number your answers correctly (especially if you're using extra sheets of paper).
- Justify your answers **fully and clearly.** Answers with no explanation (*even if the final calculation is correct*) are worth **zero** points. Answers with a full and correct explanation but a calculation error are worth more than 90% of the points.

Your Name:

Your Student ID:

1. (a) (2 points) Which of the following graphs represent real functions? Which of the functions is continuous over the interval $[-1, 1]$? Which of them has a removable discontinuity?

i. 

ii. 

(b) (3 points) Calculate the following limits.

i. $\lim_{x\to 1} \dfrac{3^x - 3}{x^2 - 1}$ ii. $\lim_{x\to 2} \dfrac{x^4 - 3x^2 - 4}{x - 2}$ iii. $\lim_{x\to \infty} \dfrac{x^2 + 4x - 3}{x^3 - 1}$

2. (a) (2 points) Calculate the derivatives of the following functions.

i. $f(x) = \tan(x^2 + 3)$ ii. $f(x) = e^{(3x^3 - \ln(x))}$

(b) (3 points) Calculate antiderivatives of the following functions.

i. $h(x) = \cos(x) \cdot e^{3\sin(x)}$ ii. $h(x) = x^2 \cdot e^x$

3. Consider the following function.

$$f(x) = \begin{cases} x^3 + 6\cos(x) \cdot \sqrt{\sin(x) + 4} & \text{if } x \geq 0 \\ 4x^2 + 12 & \text{if } x < 0 \end{cases}$$

(a) (1 point) Is f continuous at 0? Explain.

(b) (2 points) Is f differentiable at 0? Explain.

(c) (2 points) Find an antiderivative of f. (Hint: find antiderivatives when $x \geq 0$ and when $x < 0$, then put them together into a piecewise function.)

4. A company estimates their total cost function to produce x units to be

$$C(x) = 4000 + 0.25x^2 \qquad \text{thousand dollars.}$$

The company also estimates that in order to sell x units, each unit must be priced at

$$f(x) = 150 - 0.5x \qquad \text{thousand dollars.}$$

(a) (2 points) Assuming x units are produced and sold, calculate the total revenue function $R(x)$ and the total profit function $P(x)$. (Hint: total revenue = (number of units sold)·(price per unit))

(b) (2 points) How many units must be produced and sold to maximize profit? What is the maximum profit?

(c) (1 point) What price per unit must be charged to maximize profit?